

Numerical Integration

Simpson's rule:

For n even, and $h = \frac{x_n - x_0}{n}$,

$$\int_{x_0}^{x_n} f(x) \, dx \approx \frac{h}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + \dots + 2f_{n-2} + 4f_{n-1} + f_n).$$

$$\text{Truncation error} \approx -\frac{(x_n - x_0)h^4 f^{(4)}(\zeta)}{180}.$$

n point **Gauss-Legendre** formula:

$$\int_{-1}^1 f(x) \, dx \approx \sum_{i=1}^n w_i f(x_i).$$

n	x_i	w_i
2	± 0.577350	1.000000
3	± 0.774597	0.555556
	0.0	0.888889
4	± 0.861136	0.347855
	± 0.339981	0.652145
5	± 0.906180	0.236927
	0.0	0.568889
	± 0.538469	0.478629