

Particles on a slope

mc-web-mech2-7-2009

When resolving forces, the directions generally taken are horizontal and vertical. However, there are situations when it is preferential to consider other directions for resolving. Motion on a slope is one such case, where it is common practice to resolve parallel and perpendicular to the slope.

Worked Example 1.

A box, of mass 6 kg, is held at rest on a smooth slope by a force P . The slope is inclined at an angle of $\theta = 30^\circ$ to the horizontal. What is the normal reaction force between the box and the slope? (Figure 1 shows the box modelled as a particle, with the forces acting on it)

Solution

Here, as the value of R needs to be found it is more convenient to resolve perpendicular to the slope, so that only R and a component of the weight are involved in the calculations.

Hence, using Newton's Second Law of Motion:

$$\begin{aligned} F &= ma \\ R - mg \cos 30^\circ &= 0 \\ R &= 6 \times 9.81 \times \cos 30^\circ = 50.97 \\ \Rightarrow R &= 51 \text{ N (2 s.f.)} \end{aligned}$$

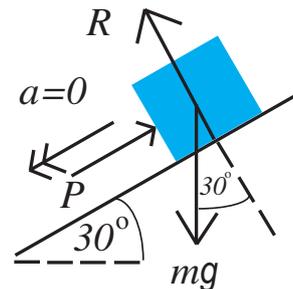


Figure 1

Worked Example 2.

A mother is out pushing her baby in its pram. She comes to a hill which is inclined at an angle of 12° to the horizontal. Assuming there is no resistance to motion and given that the pram and baby have a total mass of 11 kg, what force, P , does the mother have to push the pram with (parallel to the hill) for it to travel up the hill at a constant velocity?

Solution

As there is constant velocity this means the acceleration equals zero. As the value of P needs to be found it is more convenient to resolve parallel to the slope.

Therefore, using Newton's Second Law of Motion (parallel to the slope):

$$\begin{aligned} F &= ma \\ P - mg \sin 12^\circ &= 0 \\ P &= 11 \times 9.81 \times \sin 12^\circ = 22.44 \\ \Rightarrow P &= 22 \text{ N (2 s.f.)} \end{aligned}$$

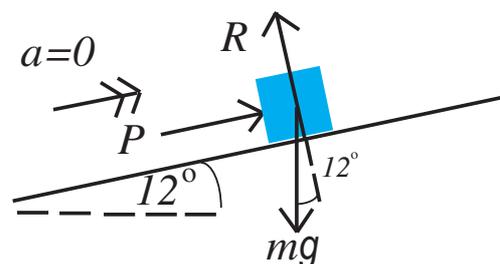


Figure 2

Exercises

1. A box is held at rest on a smooth slope. The slope is inclined at an angle of $\theta = 18^\circ$ to the horizontal. Given that the normal reaction force between the box and the slope is 79 N, what is the mass of the box?
2. A box, of mass 9 kg, is held at rest on a smooth slope, which is inclined at an angle of θ to the horizontal. Given that the normal reaction force between the box and the slope is 85 N what is the angle of the incline θ ?
3. A mother is out pushing her baby in its pram. She comes to a hill which is inclined at an angle of 8° to the horizontal and pushes the baby up the hill at a constant speed in a straight line. Assuming there is no resistance to motion and that the mother pushes with a force of 13 N parallel to the slope, what is the combined mass of the pram and baby?
4. A mother is out pushing her baby in its pram. She comes to a hill which is inclined at an angle of θ to the horizontal and pushes the baby up the hill at a constant speed in a straight line. Assume there is no resistance to motion and that the pram and baby have a combined mass of 10 kg. If the mother pushes with a force of 8.5 N parallel to the slope, what is the incline of the slope? Also, what is the normal reaction force between the pram and the ground?
5. A father pulls his child, on a sledge, up a snowy hill at a constant velocity. Assume there is no resistance to the motion and that the the child and sledge have a combined mass of 22 kg. If the hill is inclined at 20° to the horizontal and the father pulls with a force of 82 N at an angle of θ to the slope (see figure 3), find θ . What is the normal reaction force between the sledge and the hill?

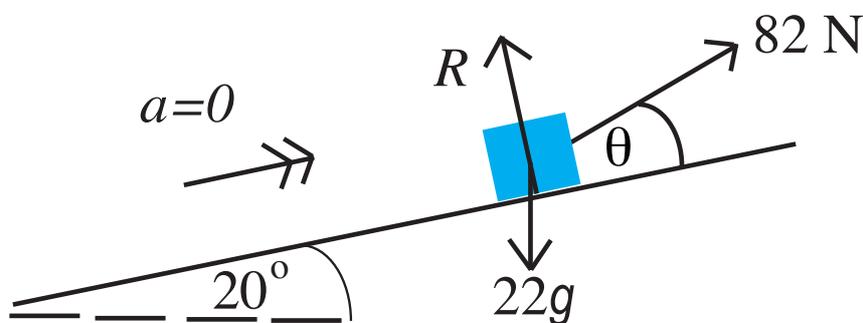


Figure 3

6. A father pulls his child, on a sledge, up a snowy hill at a constant velocity. Assume there is no resistance to the motion and that the normal reaction force is R . If the hill is inclined at 15° to the horizontal and the father pulls with a force of 45 N at an angle of 35° to the horizontal (i.e. 20° to the slope), what is the combined mass of the child and sledge?

Answers (all to 2 s.f.)

1. 8.5 kg 2. 16° 3. 9.5 kg 4. 5.0° , $R = 98$ N 5. 26° , 170 N 6. 17 kg