Ratios

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Introduction

Ratios are an alternative way of expressing fractions. This leaflet revises ratio calculations.

Ratio

Consider the problem of dividing $£200$ between two people, Ann and Bill, in the ratio $7:3$. This means that Ann receives $£7$ for every $£3$ that Bill receives. So every $£10$ is divided as $£7$ to Ann and $£3$ to Bill. So Ann receives $\frac{7}{10}$ of the money and Bill receives $\frac{3}{10}$. Now

$$\frac{7}{10} \times 200 = £140, \qquad \frac{3}{10} \times 200 = £60$$

so Ann receives $£140$ and Bill receives $£60$.

Notice how when dividing the money in the ratio $7:3$ we think of the total being made up of ten parts (7+3), with Ann being allocated seven of these parts, and Bill being allocated three.

The same is true more generally:

to divide a quantity in the ratio $m:n$ we think of the total being made up of $m+n$ parts, and split this as $\dfrac{m}{m+n}$ and $\dfrac{n}{m+n}$ of the total.

Example

Divide 170 in the ratio 3:2.

Solution

The total number of parts is $3+2=5$. We split the total as $\frac{3}{5}$ and $\frac{2}{5}$. Thus

$$\frac{3}{5} \times 170 = 102, \qquad \frac{2}{5} \times 170 = 68$$

Example

Divide \$18000 in the ratio 3:4:5.

Solution

In this example we must split the total three ways. The total number of parts is $3+4+5=12$ and the corresponding fractions are

$$\frac{3}{12}, \quad \frac{4}{12}, \quad \text{and} \quad \frac{5}{12}$$

$$\frac{3}{12} \times 18000 = 4500, \qquad \frac{4}{12} \times 18000 = 6000, \qquad \text{and} \qquad \frac{5}{12} \times 18000 = 7500$$

The simplest form of a ratio

A ratio remains unchanged if each of its constituent parts is multiplied or divided by the same number.

So, for example, the ratio

$$3 : 5 : 8 \qquad \text{is the same as} \qquad 6 : 10 : 16$$

Similarly

$$\frac{5}{4} : \frac{2}{3} \quad \text{is the same as} \quad 5 : \frac{8}{3} \qquad \text{(by multiplying by 4)}$$

which is also the same as

$$15 : 8 \qquad \text{(by multiplying the last result by 3)}$$

Increasing quantities in a given ratio

Suppose we are asked to increase $\pounds 60$ in the ratio $8 : 5$. What this means is that every $\pounds 5$ is increased to $\pounds 8$. Now there are $\frac{60}{5} = 12$ lots of $\pounds 5$ in $\pounds 60$. If each is increased to $\pounds 8$ the total amount will then be $12 \times 8 = \pounds 96$.

This calculation is the same as

$$\frac{8}{5} \times 60 = 96$$

In general to increase a quantity Q in the ratio $m : n$ we calculate

$$\frac{m}{n} \times Q$$

If m is less than n then the quantity will be decreased.

Example

Decrease 1025 in the ratio $3 : 5$.

Solution

$$\frac{3}{5} \times 1025 = 615$$

Exercises

1. Cartridge brass has a ratio of copper to zinc of 7:3. Calculate the mass of the metallic constituents in 50kg of cartridge brass.

2. Express the ratio $\frac{1}{3} : 2$ in its simplest form.

3. Increase 450 in the ratio 3:2.

Answers

1. 35kg copper, 15kg zinc. 2. 1:6 3. 675