

Interest

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Introduction

Interest rate calculations arise in a variety of business applications, and affect all of us in our personal and professional lives. Individuals earn interest on sums they have invested in savings accounts. Many home owners pay interest on money they have borrowed for mortgages, personal loans etc. This leaflet revises interest and its calculation.

Simple interest

An amount of money which is invested, or borrowed, is called the **principal**. The amount of interest which is earned, or paid, depends upon the principal, the period of time over which the interest can accumulate, and the **interest rate**. The interest rate is usually expressed as a percentage per period of time, for example 6% per annum.

The simplest form interest can take is called **simple interest** and is given by the formula:

$$I = P i n$$

where I is the interest earned or paid, P is the principal, i is the interest rate per time period, and n is the number of time periods over which the interest accumulates.

Example

If an amount of £4000 is invested in a savings account at an interest rate of 8% per year, calculate the (simple) interest paid over a three year period.

Solution

Here $P = £4000$, $i = 8\% = \frac{8}{100} = 0.08$, and $n = 3$. So

$$\begin{aligned} I &= P i n \\ &= (4000)(0.08)(3) \\ &= £960 \end{aligned}$$

This is equivalent to earning interest of £320 in each of the three years.

In the previous example we can imagine earning interest of £320 each year, and withdrawing it immediately it is paid. However, in practice the interest earned at the end of each year can be left in the savings account so that it too earns interest. This leads to the concept of **compound interest**.

Compound interest

In compound interest calculations interest earned, or due, for each period, is added to the principal.

Example

Repeat the previous example assuming that interest is compounded at the end of each year.

Solution

At the end of year 1 the interest earned is £320. This is added to the existing principal to give £4320.

At the end of year 2 the interest earned is

$$I = (4320)(0.08)(1) = £345.60$$

Note that more interest is earned in the second year because of compounding. At the end of year 2 this is added to the existing principal to give £4320 + £345.60 = £4665.60.

At the end of year 3 the interest earned is

$$I = (4665.60)(0.08)(1) = 373.25$$

Hence the final accrued amount is £4665.60 + £373.25 = £5038.85. This represents a total interest payment of £1038.85. Compare this with the amount of simple interest earned over the same time period.

Compound amount formula:

To avoid the lengthy calculation above a formula is available for use in compound interest calculations.

If a principal P is invested at a rate i per time period, it accrues to an amount S after n time periods given by

$$S = P(1 + i)^n$$

Example

Use this formula to calculate the amount to which £4000 grows at an interest rate of 8% per annum, for three years, if all the interest earned is reinvested.

Solution

Here the interest rate is $8\% = \frac{8}{100}$ or 0.08.

$$\begin{aligned} S &= P(1 + i)^n \\ &= 4000(1 + 0.08)^3 \\ &= £5038.85 \end{aligned}$$

confirming the result of the previous Example.

Exercise

An investment of £300,000 is made at an annual interest rate of 5.5%. If interest is compounded for a period of 10 years calculate the total interest earned and the accrued amount after the ten year period.

(Answer: £212,443, £512,443.34).