

## Algebra

### Removing brackets:

$$a(b + c) = ab + ac, \quad a(b - c) = ab - ac$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

### Formula for solving a quadratic equation:

$$\text{if } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Laws of Indices:

$$a^m a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \quad (a^m)^n = a^{mn}$$

$$a^0 = 1 \quad a^{-m} = \frac{1}{a^m} \quad a^{1/n} = \sqrt[n]{a} \quad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

### Laws of Logarithms:

$x = \log_a y$  means  $a^x = y$  and  $a$  is called the **base**.

e.g.  $\log_{10} 2 = 0.3010$  means  $10^{0.3010} = 2.000$ , to 4 s.f.

In any base

$$\log AB = \log A + \log B, \quad \log \frac{A}{B} = \log A - \log B,$$

$$\log A^n = n \log A, \quad \log 1 = 0$$

Logarithms to base e, denoted  $\log_e$  or alternatively  $\ln$  are called *natural logarithms*. The letter e stands for the exponential constant which is approximately 2.718.

## Proportion and Percentage

To convert a fraction to a percentage multiply by 100 and label the result as a percentage.

### Examples

$\frac{5}{8}$  as a percentage is  $\frac{5}{8} \times 100\% = 62.5\%$

$\frac{1}{3}$  as a percentage is  $\frac{1}{3} \times 100\% = 33\frac{1}{3}\%$

Some common conversions are

$$\frac{1}{10} = 10\%, \quad \frac{1}{4} = 25\%, \quad \frac{1}{2} = 50\%, \quad \frac{3}{4} = 75\%$$

**Ratios** are simply an alternative way of expressing fractions. Consider dividing £200 between two people in the ratio of 3:2. This means that for every £3 the first person gets, the second person gets £2. So the first gets  $\frac{3}{5}$  of the total, and the second gets  $\frac{2}{5}$  of the total; that is £120 and £80.

Generally, to split a quantity in the ratio  $m : n$ , the quantity is divided into  $\frac{m}{m+n}$  of the total and  $\frac{n}{m+n}$  of the total.