

The complex conjugate

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In this unit we are going to look at a quantity known as the **complex conjugate**.

Every complex number has associated with it another complex number known as its complex conjugate. You find the complex conjugate simply by changing the sign of the imaginary part of the complex number.

Example

To find the complex conjugate of $4+7i$ we change the sign of the imaginary part. Thus the complex conjugate of $4+7i$ is $4-7i$.

Example

To find the complex conjugate of $1-3i$ we change the sign of the imaginary part. Thus the complex conjugate of $1-3i$ is $1+3i$.

Example

To find the complex conjugate of $-4-3i$ we change the sign of the imaginary part. Thus the complex conjugate of $-4-3i$ is $-4+3i$.

The complex conjugate has a very special property. Consider what happens when we multiply a complex number by its complex conjugate.

For example, multiplying $(4+7i)$ by $(4-7i)$:

$$\begin{aligned}(4+7i)(4-7i) &= 16 - 28i + 28i - 49i^2 \\ &= 16 + 49 \\ &= 65\end{aligned}$$

We find that the answer is a purely real number - it has no imaginary part. This always happens when a complex number is multiplied by its conjugate - the result is real number.

Example

$$\begin{aligned}(1-3i)(1+3i) &= 1 + 3i - 3i - 9i^2 \\ &= 1 + 9 \\ &= 10\end{aligned}$$

Once again, we have multiplied a complex number by its conjugate and the answer is a real number. This is a very important property which applies to every complex conjugate pair of numbers. We will use this property in the next unit when we consider division of complex numbers.