Engineering Maths First Aid Kit

6.1

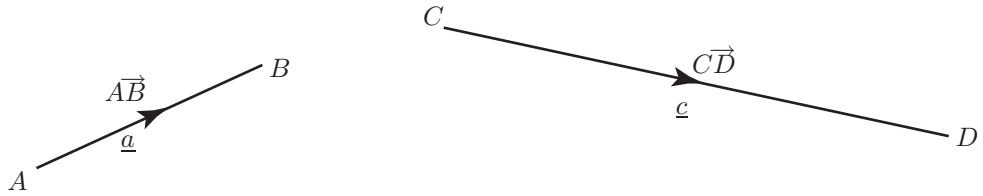
Vectors

Introduction

This leaflet explains notations in common use for describing vectors, and shows how to calculate the modulus of vector given in cartesian form.

1. Vectors

Vectors are quantities which possess a **magnitude** and a **direction**. As such, we often represent them by **directed line segments** such as those shown below.



The arrow on the line indicates the intended direction whilst the length of the line represents the magnitude. The magnitude is also called the **modulus** or the **length** of the vector.

It is important when writing vectors that we distinguish them from scalars (or numbers) and so various notations are used to do this. We can write the vector from A to B as $\overrightarrow{AB}$. In printed work vectors are often shown with a bold typeface, as in $\mathbf{a}$. In handwritten work we usually underline vectors, as in $\underline{a}$. Whichever way you choose it is important that vectors can be distinguished from scalars. The magnitude of a vector $\underline{a} = \overrightarrow{AB}$ is written as $|\underline{a}|$ or $|\overrightarrow{AB}|$. The magnitude is represented by the length of the directed line segment.

2. Unit vectors

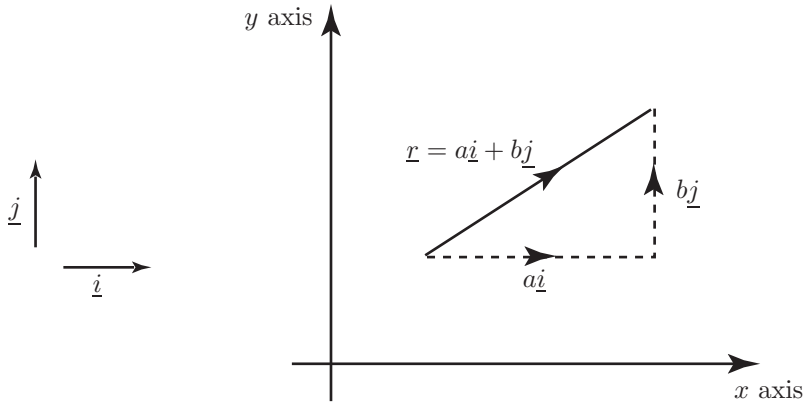
A **unit vector** is a vector of length 1. To obtain a unit vector in the direction of any vector $\underline{a}$ we divide by its modulus. To show a vector is a unit vector we give it a 'hat', as in $\hat{\underline{a}}$.

$$\hat{\underline{a}} = \frac{\underline{a}}{|\underline{a}|}$$

3. Cartesian components

$\underline{i}$ represents a unit vector in the direction of the positive x axis

$\underline{j}$ represents a unit vector in the direction of the positive y axis



Any vector in the xy plane can be written $\underline{r} = a\underline{i} + b\underline{j}$ where a and b are numbers.

Its modulus can be found using Pythagoras' theorem:

$$|\underline{r}| = \sqrt{a^2 + b^2}$$

4. Three dimensions

To work in three dimensions we introduce an additional unit vector $\underline{k}$ which points in the direction of the positive z axis.

Any vector in three dimensions can be written $\underline{r} = a\underline{i} + b\underline{j} + c\underline{k}$.

Its modulus can be found using Pythagoras' theorem:

$$|\underline{r}| = \sqrt{a^2 + b^2 + c^2}$$