Factorising simple expressions

Introduction

Before studying this material you must be familiar with the process of 'removing brackets' as outlined on leaflets 2.3 & 2.4. This is because factorising can be thought of as reversing the process of removing brackets. When we factorise an expression it is written as a product of two or more terms, and these will normally involve brackets.

1. Products and Factors

To obtain the **product** of two numbers they are <u>multiplied</u> together. For example the product of 3 and 4 is 3×4 which equals 12. The numbers which are multiplied together are called factors. We say that 3 and 4 are both factors of 12.

Example

The product of x and y is xy.

The product of $5x$ and $3y$ is $15xy$.

Example

$2x$ and $5y$ are factors of $10xy$ since when we multiply $2x$ by $5y$ we obtain $10xy$.

$(x+1)$ and $(x+2)$ are factors of $x^2 + 3x + 2$ because when we multiply $(x+1)$ by $(x+2)$ we obtain $x^2 + 3x + 2$.

3 and $x - 5$ are factors of $3x - 15$ because

$$3(x-5) = 3x - 15$$

2. Common Factors

Sometimes, if we study two expressions to find their factors, we might note that some of the factors are the same. These factors are called **common factors**.

Example

Consider the numbers 18 and 12.

Both 6 and 3 are factors of 18 because $6 \times 3 = 18$.

Both 6 and 2 are factors of 12 because $6 \times 2 = 12$.

So, 18 and 12 share a common factor, namely 6.

In fact 18 and 12 share other common factors. Can you find them ?

Example
The number 10 and the expression $15x$ share a common factor of 5.

Note that $10 = 5 \times 2$, and $15x = 5 \times 3x$. Hence 5 is a common factor.

Example
$3a^2$ and $5a$ share a common factor of a since

$3a^2 = 3a \times a$ and $5a = 5 \times a$. Hence a is a common factor.

Example
$8x^2$ and $12x$ share a common factor of $4x$ since

$8x^2 = 4x \times 2x$ and $12x = 3 \times 4x$. Hence $4x$ is a common factor.

3. Factorising

To factorise an expression containing two or more terms it is necessary to look for factors which are common to the different terms. Once found, these common factors are written outside a bracketed term. It is ALWAYS possible to check your answers when you factorise by simply removing the brackets again, so you shouldn't get them wrong.

Example
Factorise $15x + 10$.

Solution
First we look for any factors which are common to both $15x$ and 10. The common factor here is 5. So the original expression can be written

$$15x + 10 = 5(3x) + 5(2)$$

which shows clearly the common factor. This common factor is written outside a bracketed term, the remaining quantities being placed inside the bracket:

$$15x + 10 = 5(3x + 2)$$

and the expression has been factorised. We say that the factors of $15x + 10$ are 5 and $3x + 2$. Your answer can be checked by showing

$$5(3x + 2) = 5(3x) + 5(2) = 15x + 10$$

Exercises
Factorise each of the following:

1. $10x + 5y$,   2. $21 + 7x$,   3. $xy - 8x$,   4. $4x - 8xy$

Answers
1. $5(2x + y)$,   2. $7(3 + x)$,   3. $x(y - 8)$,   4. $4x(1 - 2y)$.