Mathematical Sciences HE Curriculum Innovation Project

HE Mathematics Curriculum Summit

A report on the summit held at the University of Birmingham on 12 January 2011

Edited by Peter Rowlett

The Higher Education Academy Maths, Stats & OR Network

National HE STEM Programme

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Organised under the Mathematical Sciences HE Curriculum Innovation Project by the Maths, Stats and OR Network as part of the Mathematical Sciences Strand of the National HE STEM Programme.

Edited by Peter Rowlett

March 2011

The Higher Education Academy Maths, Stats & OR Network

National HE STEM Programme

Summit participants

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University	Representative
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University of Birmingham	Dr Chris Sangwin
University of Cambridge	Dr Stephen Siklos
Cardiff University	Dr Robert Wilson
University of Chester	Dr Yubin Yan
University of Derby	Paul Dale
University of Exeter	Prof Peter Ashwin
University of Greenwich	Prof Kevin Parrott
University of Hertfordshire	Dr Stephen Kane
Kingston University	Dr Nigel Atkins
University of Leeds	Dr Margit Messmer
University of Leicester	Prof Jeremy Levesley
London Metropolitan University	Dr Pargat Calay
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University of Manchester	Dr Louise Walker
Newcastle University	Dr Bill Foster
Nottingham Trent University	Dr James Hind
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University of Plymouth	Dr David Graham
University of Portsmouth	Prof Andrew Osbaldestin
University of Reading	Dr Nick Biggs
University of Salford	Dr Edmund Chadwick
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Organisation	Representative
Council for the Mathematical Sciences	David Youdan
Institute of Mathematics and its Applications	Prof Nigel Steele
National HE STEM Programme	Michael Grove
The OR Society	Dr Stuart Johns
Royal Statistical Society	Dr Penelope Bidgood

Individuals

Name	Organisation
Prof Alexandre Borovik	University of Manchester
Jon McLoone	Wolfram Research
Prof Tony Croft	MSOR Network
Dr Chris Good	MSOR Network
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Dagmar Waller	MSOR Network
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Foreword

The National HE STEM Programme is funded by the Higher Education Funding Councils for England and Wales and seeks to support Higher Education Institutions in encouraging the exploration of new approaches to recruiting students and delivering programmes of study. It enables the transfer of good practice across the HE STEM sector, facilitates its wider adoption, and encourages innovation. Through collaboration and shared working, the Programme focuses upon sustainable activities to achieve long-term impact within the HE sector.

The IMA is leading the Mathematical Sciences Strand of the National HE STEM Programme and is assisted on *HE curriculum innovation* by the MSOR Network. The HE curriculum innovation activities explore current learning, teaching and assessment practices within mathematical sciences departments, and disseminate good practice. This component fits into a wider programme of activity in mathematical sciences, where the IMA is working on *integration and diversity, employer engagement* and, with sigma, on *mathematical sciences support*.

The Mathematics HE Curriculum Summit in January 2011, run by the MSOR Network as part of the HE curriculum innovation activity, attracted a wide and representative cross-section of the HE mathematical sciences community. One measure of the success of the event is the range of participants from our community and another is the enthusiasm with which they engaged with identifying priorities for research and development. I am confident that our work in HE curriculum innovation will be relevant to the needs of the community it looks to serve, being well informed by the views of that community. I hope this report of the findings from the Summit will also serve as a record of current issues and priorities for future development of the HE Mathematics Curriculum.

M Walker

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1. Introduction

The HE Mathematics Curriculum Summit took place at the University of Birmingham on 12 January 2011, operated by the Maths, Stats and OR (MSOR) Network as part of the Mathematical Sciences HE Curriculum Innovation Project within the National HE STEM Programme. This brought together: Heads of Mathematics or their representatives from 26 universities offering mathematics degrees (about half of those in England and Wales); Education representatives from the Institute of Mathematics and its Applications, the Royal Statistical Society, the Operational Research Society and the Council for the Mathematical Sciences; members of the National HE STEM Programme, sigma and the MSOR Network; and several individuals.

The day was chaired by Prof. Duncan Lawson and opened with a debate, in which Prof. Alexandre Borovik of University of Manchester proposed and Jon McLoone of Wolfram Research opposed the motion ‘We believe that memory, subject knowledge and technical fluency remain vital for undergraduate mathematicians in the digital age’. Following this, breakout groups discussed the topics: ‘We can’t let them graduate unless...’; ‘If maths students can’t communicate in writing or speak in public – is that my problem?’; and, ‘If most maths graduates “aren’t confident” in handling unfamiliar problems – should we care?’ After lunch the Summit received feedback from the morning discussions and an update on employer engagement activity from the Mathematical Sciences Strand by David Youdan. The Summit heard and discussed presentations from Prof. Jeremy Levesley on ‘Taking control of the assessment agenda’ and Dr. Neil Challis on ‘What do the students think about their Maths degrees?’ A final set of breakout sessions considered the topic: ‘Imagine there is £100k-£150k in total available to support curriculum development across the sector, how best should this be targeted and what are the priority areas?’

This document contains reports on the debates, presentations and discussions held at the Summit and a summary of the recommendations made in the final discussion groups for priority activities in HE mathematics curriculum development. As well as being a record of current sector priorities, these recommendations will be considered when planning activities for the Mathematical Sciences HE Curriculum Innovation Project.

I am grateful to those who attended the Summit for taking the time to contribute to these discussions. Thanks also to Prof. Alexandre Borovik, Jon McLoone, Prof. Jeremy Levesley and Dr. Neil Challis for contributing sessions to the Summit. Thanks to Prof. Duncan Lawson, Prof. Tony Croft, Dr. Chris Sangwin, Dr. Chris Good, Dagmar Waller, Moira Petrie, Dr. Joe Kyle, Efua Wilson-Tagoe and Janet Nuttall for helping run the day and taking notes for this report. This report has benefited from comments made by Dr. Chris Sangwin and Prof. Duncan Lawson.

Peter Rowlett, March 2011

2. Debate

‘We believe that memory, subject knowledge and technical fluency remain vital for undergraduate mathematicians in the digital age’

Report of a debate between Alexandre Borovik and Jon McLoone

by Dr Joe Kyle

The Summit meeting opened with a formal debate between Alexandre Borovik, University of Manchester and Jon McLoone, Wolfram Research, on the motion ‘We believe that memory, subject knowledge and technical fluency remain vital for undergraduate mathematicians in the digital age’. Before the protagonists opened the discussion Prof. Duncan Lawson conducted a poll to determine the “initial state” of opinion among the audience.

First to speak was Prof Borovik who, typical of a pure mathematician, set out his definitions and assumptions including the beguiling assertion that mathematics is not what it seems to be. For Borovik, memory in mathematics was about remembering links and relations between mathematical facts, and even more: relations between relations. But perhaps the most original contribution was Borovik’s assertion of the “Law of Excessive Learning of Mathematics”: to be able to use mathematics successfully at one level, it is necessary to activate it by using it in learning at the next level. For this speaker, technical fluency was the ability, not necessarily to use and understand mathematical formula, but to parse mathematical objects: to read and see the structure therein. According to this view, when students are performing technical manipulations they are learning not to write mathematics, they are learning to read. Even in a digital age, the need to have the ability to parse mathematical objects will not disappear.

By the same token, real progress through the hierarchy of mathematical structures is only possible if lower order mathematical objects and structures have been “interiorized” as essential scaffolding. To advance to another level requires an ongoing process of exercising technical fluency at lower levels in a repeated cycle of encapsulation, de-encapsulation and re-encapsulation. This, according to Borovik, was a basic cognitive mechanism that had developed through social and biological evolution. How does IT-mediated learning fit in with this model or even possibly help it to develop? In posing these questions Borovik concluded his talk and at the same time provided a neat link to the next speaker.

Jon McLoone’s basic premise was that we should move away from the more traditional view of subject knowledge and technical fluency and wherever possible use computers, both in the use and teaching of mathematics. Addressing the question “Why teach Maths?”, McLoone offered three answers: the ongoing supply of analytic problems (from industry), the increasingly technical nature of ordinary living, and the need to develop logical thinking.

As with the first speaker, Jon McLoone also offered some definitions to place his argument in context and his first was a definition of mathematics. The practice of mathematics involves four stages: posing the right problem, converting the real world into a mathematical formulation, computation, re-interpreting back in the real world. (It should be mentioned here that in discussion this definition came in for some criticism in that it defined mathematics very much as a modelling cycle.) His case was that we should harness computing for the third stage, computation, thereby freeing up human creativity for the other three for which computers were not (as yet) particularly good. McLoone’s position on memory did not differ greatly from his opponent: the use of memory for simply remembering facts is not really the main concern of mathematics. This point was illustrated with some amusing examples demonstrated, of course, on Mathematica. In discussing technical fluency, McLoone also agreed with much in Borovik’s talk. However, in deciding which areas required technical fluency, McLoone identified two features as important: that lack of human fluency would create a “bottleneck” in mathematics (as defined above) or, it was needed for understanding. McLoone agreed that subject knowledge would always be needed, but queried whether we were making the best use of time. Asserting that “over 100 lifetimes” are today devoted to practising hand-calculation, he made a plea for using that time for more “mind-expanding” opportunities in areas such as intuition, connections between various disciplines, open-ended problems but above all, the fun in mathematics.

A number of interesting issues emerged during the open session when the debate was thrown open to all participants. The early questions were addressed to Borovik and concerned the relative importance of the attributes discussed and the role of memory. Borovik replied that in his view a number of skills were temporary skills, but skills needed at the time. He also saw abstract thinking as a more important attribute and illustrated his reply with anecdotes from his own education adding that he did not like the idea that a person’s memory required an external power source - summing up to laughter: “[so if] there is no electricity, there is no mathematics”.

One questioner put it to McLoone that, despite seeing advantages to the use of technology, there seemed to be evidence that this did not always bring with it understanding, citing the effects of hand-held calculators. In reply, McLoone said he thought this was due to a systemic problem in primary and secondary education. Another similar comment put the case that much that was now being claimed about computers had been asserted by those advocating calculators in the 1970s, and that in both were overlooking the essential pedagogic precept that learning mathematics was a “messy” hands-on process. One member of the audience pointed out that the speakers had defined everything but the term “undergraduate mathematician” and yet this term covered a very diverse group in the UK, while another contribution highlighted the need to look to serving the needs of the graduate seeking employment. Invited to address directly Borovik’s Law of Excessive Learning, McLoone pointed out that while it had its place, sometimes the desire to use something was greater than the ability to compute it and then a computational aid could play a useful role. One participant saw limitations in the Law of Excessive Learning when it came to the development of new areas or the creation of new tools: if the next level up has not yet been created, how does the Law apply?

The debate concluded with a re-run of the initial poll. It turned out that the net change was a swing of one vote from the Borovik camp to the McLoone camp (25 for, 6 against before the debate; 24 for, 7 against after), and it is almost certain that this was due to a change of vote by precisely one person!

To this observer, this debate was a success. Two powerful speakers put their arguments with clarity, conviction and a degree of humour. Admittedly, there was a feeling that the principal speakers worked hard to create the impression of disagreement when one felt there was no great division in principle. However, the debate achieved the primary purpose of stimulating discussion, debate and interaction and provided the ideal platform for the rest of the meeting. For this both speakers deserve thanks for providing thought-provoking presentations and leading an engaging discussion.

3. Discussion group reports

I. 'We can't let them graduate unless...'

by Dr. Chris Good

Discussion group chair: Dr. Chris Sangwin.

Initially, the group addressed the topic without much hope of coming to a consensus. Two of the members had been involved in previous attempts at benchmarking and reported the difficulties of this process which agreed on none or very little common essential content. However, the discussion reached considerable consensus. The results of the discussion agreed in large part with the QAA Mathematics, statistics and operational research Benchmark statement [1].

The discussion topic could be usefully approached in several ways. For example: what specific skills and mathematical knowledge do employers expect of mathematics graduates? What would it embarrass university mathematicians to find out their students did not know or, at least, had not been exposed to?

The answer depends in large part on the university and degree programme in question. Mathematics is not a vocational subject, although mathematics graduates are sought after by employers. The content of a degree programme will and should change with time, fashions within the subject and societal need. An example here is the difference in popularity of Euclidean Geometry in mathematics education now compared with one hundred years ago.

Despite this variety, a degree programme should offer students the (somewhat nebulous) idea of what it is to be a mathematician and the possibility of becoming one. (Although the group wondered how many undergraduates would be able to articulate what it meant to be a mathematician.) In particular, students should realise that mathematics is not just a formulaic process. Following Borovik's 'Law of Excessive Learning', graduates of a mathematics degree should have been exposed to enough mathematics to become a school mathematics teacher. Furthermore, a mathematics degree should challenge its students.

Each member of the group undertook to list the top five topics they felt a mathematics undergraduate must not graduate without knowing. Mathematics is regarded as separate from statistics and OR but as the focus of the discussion was on graduates of mathematics degrees it did not exclude knowledge of statistics and OR. The responses showed some considerable agreement. The results of this exercise are summarised below.

Graduate attributes:

- Problem solving;
- Flexibility;
- The ability (and desire) to learn more and go further;
- Enthusiasm for the subject;
- Some knowledge of the culture of mathematics;
- The ability to communicate mathematics.

Subject knowledge:

A mathematics degree should provide some knowledge of:

- Linear algebra and its applications;

- The foundations and applications of calculus and analysis;
- The need for proof and techniques of proof;
- Modelling;
- Probability and statistics;
- Differential equations and their use;
- Abstract algebra and its applications.

Specific topics that were mentioned in the lists are below. N.B. one or two of the answers were provided in full awareness of the exercise and were perhaps somewhat tongue-in-cheek. The constraints of time meant the group did not review this list after it was compiled or discuss its contents in any great depth. There were 14 participants in this exercise.

- The notion of limit/continuity, Elementary Analysis, Calculus and its Foundations (9 votes)
- Linear Algebra (8 votes)
- Techniques of proof (5 votes)
- Differential Equations and their use (5 votes)
- Modelling (of some sort) (4 votes)
- Basic statistical tests (2 votes)
- Probability/an understanding of risk (2 votes)
- $e^{i\pi}$ and its consequences (1 vote)
- 1 vote each: The infinitude of the primes; The irrationality of the square root of 2; The proof of Pythagoras's theorem; Fundamentals of Formal Logic
- Numerical Methods (1 vote)
- 1 vote each: Programming; Applications of appropriate software
- Graph Sketching (1 vote)
- 1 vote each: Vector Calculus; Fourier Analysis; Special Functions
- 1 vote each: Abstract Algebra/Group Theory; Number Theory

References

1. The Quality Assurance Agency for Higher Education, 2007. Mathematics, statistics and operational research.

II. ‘If maths students can’t communicate in writing or speak in public – is that my problem?’

by Dagmar Waller and Peter Rowlett

Discussion group chair: Prof. Duncan Lawson.

Addressing skills development is the concern of mathematics educators. Many students take a mathematics degree not because they are keen on maths but in order to get a good job. Employability is a major factor for students in choosing where to take their degree so there is a need to respond to attract potential students. Those offering mathematics degrees have a duty and a necessity to meet students’ career aspirations, which includes developing graduate skills. It may be unfair to graduates to have degree courses that don’t offer the chance to experience giving presentations and other communication skills activities.

However, some students choose mathematics specifically to get away from having to present through long form written and oral communication. There is some suggestion students who are attracted to maths are likely to have low social ability and this will affect their ability to communicate effectively. However, students are not given the choice to opt-out on certain mathematical content and they shouldn’t be given this option with learning to communicate. In order to manage expectations, it is important that students are told up front to expect skills development during their course. Students should be told about employers’ expectations and the place such skills will have in their careers. Communication skills are a small part of what employers want but are essential. In industry the person who is able to give a presentation to communicate their ideas is the one with the power. Mathematics graduates are often working in the background and someone else may attract credit for their work.

Students must be taught to write mathematics precisely and to incorporate mathematics into sentences. There is more to writing a good mathematical report than just grammar. As well as issues with students’ use of language, students must be able to effectively plan what they want to say. Hand writing mathematics and presenting mathematical work in a logical way are important skills. Typesetting mathematics can also be an important communication skill. Students of other subjects are expected to be able to present their work in both hand written and typeset formats. The ability of mathematicians to communicate with non-mathematicians is an important skill. There is also an issue with private communication within small groups of students, for example not talking to each other when working through problems. Students are not used to talking to each other about mathematics, believing it to be a solitary subject. When students work collectively and help each other – as in maths support centres – they are more involved and engaged. There is an issue of deciding where this should be developed and how much formal opportunity to work in this way should be provided. There is also an issue of drawing a careful distinction between working constructively together and plagiarism.

At present, most mathematics degree courses do not include such skills development. Training in writing and presenting can be very resource intensive to deliver. However, most courses have a dissertation, which requires a level of written communication ability, so there is a need to put skills development in place in the first two years of a programme to prepare students for the final year. Although communication skills are very important, the penalty in a piece of work that otherwise demonstrates a high level of mathematical knowledge and ability should not be so severe that the student will fail the assessment on poor communication skills alone.

Although most degrees do not include this very basic skills development, employer groups such as the CBI would prefer skills development to be at a higher level and should, for example, incorporate team building and project management skills. It is important for universities to respond to the priorities of such groups and there is evidence from physical sciences that students want more support in group working and project management.

The ability to deliver skills development may lie outside of the expertise of the mathematics specialists. However, the need to communicate is not a generic one; there is a need for students to communicate on mathematical subjects, especially to non-mathematicians. Many members of staff are not confident in the use of software and technology needed to communicate effectively and, for example, do not include technology in their teaching. This reluctance to engage with new technologies can undermine the message presented to students regarding the importance of quality communication. There is a need to develop staff confidence in this area and staff must be given the time away from other duties to allow them to develop their teaching skills. Learning to deliver skills development should improve the skills of lecturers as well.

There is a question of whether skills should be delivered through teaching or through the opportunity to practice. It is possible to allow the students to practice and provide feedback on the results, although if communication skills are so valued it may not be enough to let the students just try it without any extra training. If students are provided with regular opportunities to practice their skills, improvements should be observed, although such opportunities and associated feedback can be resource intensive to deliver.

Having stressed the importance of skills within mathematics, it is important to remember that university is about education and not just training; we are not offering a degree in employability and the study of mathematics for the advancement of knowledge must be remembered. However, students have many reasons for coming to university and there are challenges ahead in a changing world. There should be room within a degree in mathematics to develop employability skills.

Taking time for skills development does not necessarily mean less maths can be covered or ‘dumbing down’ the technical content of degrees if skills development is embedded in courses and delivered through mathematical content. Examples of useful approaches are: year 1 presentations in first week; a modelling week each year; group working; modelling; student presentations on mathematical topics they are covering in their modules. Development of skills in this embedded way requires the same breadth of subject ability.

When considering the development of skills in an embedded way, note that current mathematics teaching may not be fit for purpose. Traditional teaching is orientated to a world that no longer exists; for example, it tends to be focused on physical sciences. There is a tendency for mathematicians to focus on content without a clear idea of what can be done with it or where it can be applied. Mathematics graduates are not necessarily equipped to operate effectively using mathematical training in the real world. There needs to be a more flexible approach to meet employer need.

There is no need for all degree programmes to look the same and it is helpful for student choice if there are degrees which offer differing levels of focus on employability. Institution-wide initiatives may impact on this issue in individual departments.

III. ‘If most maths graduates “aren’t confident” in handling unfamiliar problems – should we care?’

by Moira Petrie and Peter Rowlett

Discussion group chair: Prof. Tony Croft.

It is possible that confidence in handling unfamiliar problems comes with maturity and work-based learning. If this is the case, there may only be so far that the HE curriculum can go. Still, the issue of graduates’ confidence in unfamiliar problems should concern HE mathematics educators.

When mathematics degree courses are advertised to students, adaptability and unfamiliarity are usually key selling points, making confidence with unfamiliar problems part of the added value of a mathematics degree. The question of unfamiliar problems is linked to issues around employability. In many areas of employment, graduates will not be tackling familiar problems, which can often be automated, but will be tackling unfamiliar problems. Many graduates may not use their mathematics skills in their jobs but confidence in problem solving is a skill that certainly will be useful. Problem solving is the most useful skill a student can take with them when they leave university. It is problematic to allow students to graduate with first class degrees who cannot handle unfamiliar problems.

However, there is both student and staff resistance to unfamiliar problem solving. Current teaching and assessment methods do not tend to develop these skills and may need to change. It is important that those taking a mathematics education are made to realise how little they actually know.

A different approach to teaching may be required in order to develop skills and confidence in unfamiliar problems. Students can be focused on learning and applying methods and a change is needed so that they begin to think creatively for themselves. One way to develop this sort of teaching is through lecturers acting as role models. Students need to see lecturers trying to solve unfamiliar problems, particularly ‘dirty’ problems with substantial risk of failure.

There is a natural ability element to problem solving and this may cause issues when trying to teach this skill. It may only be possible to give the students the opportunity to practice and to give them effective feedback and the encouragement that it is okay to fail as long as they keep trying. This approach should build confidence. Confidence in solving unfamiliar problems is best developed in a group with shared discourse, rather than individually.

Current assessment methods may encourage rote learning, which doesn’t encourage deep understanding and adaptability to unfamiliar problems. These methods need to change to enable assessment of ability to handle unfamiliar problems.

The subdivision of the subject (modularisation) is a barrier to solving problems in unfamiliar areas, many of which will not be tightly-focused. Students may not understand the links between modules, seeing them as independent and stand alone. Confidence in handling unfamiliar problems must be tested in the right setting. Traditional examinations are not the right setting so alternatives need to be explored. Open ended problems can be difficult and time-consuming to mark and this is a particular issue with requirements for short turnaround times for feedback.

Students may expect problems to have neat solutions and single best approaches. Students may like getting to an answer, knowing when they are finished and whether they are correct. Unfamiliar problems may not fit this profile. Many students choose mathematics at university because it’s a safe option and has correct answers (at A-Level). Some students refuse to look at unfamiliar problems.

Unfamiliar problems carry an increased risk of failure for the students and such failure may be an important part of the learning process. Since students are often assessment driven, and every assessment contributes to the overall grade, high achieving students in particular do not like to fail. Students must be able to ‘fail’ an assessment to learn from trial and error and develop a good critical thought process.

4. ‘Taking Control of the Assessment Agenda’

by Prof. Jeremy Levesley

Higher Education is sailing into very difficult waters. We will need to demonstrate value for the large fee that will be paid. In order to do this we must be very clear about what our degrees deliver, and we need to be able to assess this. Since we will not want to increase the amount of assessment that we do, we will need to find efficient modes of testing, which provide useful and timely feedback for the student body. It is my belief that the mathematics community knows best how to do this, and that student learning, and our subsequent teaching and assessment practices should be designed by us, in light of what we wish our degrees to deliver. In particular we should not be being driven by external quality management requirements.

I believe our main obstacle is conservatism inside the mathematics sector, and risk averseness in the quality management function of universities, who are anxious about QAA inspections.

The key points for assessment I believe are:

1. Assessment should be aspirational for students at all stages; students should be able to get a first class degree right until the end of the final year. Any other scheme means that students will decide not to try because they are restricted in the final outcome.
2. Students should be being assessed near to the edge of their competence. Too much assessment is repetitive, testing well below the ability of many of the students.
3. We need to be very clear about how competences match to levels. Students get 2ii degrees because they cannot do difficult things well, not because they can do more basic things adequately.
4. We need to decide on how many times a competence has to be demonstrated before the student is said to be able to do it. In this way, students can progress through the levels of a scheme. If we do this then we can unhitch degree classification from module performance, and attach it to demonstration of competence shown across the degree.
5. Students should view assessment as their opportunity to demonstrate their competences to us, not as a thing that we do to them.
6. We need to collect the appropriate pedagogy to justify progression in and assessment of competencies. We rely too much on opinion and anecdote.
7. Action across the mathematics community together in this will give us a greater chance to influence our institutions, and the QAA.

What I say above relates also to formative assessment. The more opportunity for self and peer assessment that the students have in order to get to know themselves, the better.

Of course, Points 1- 6 above lead to a number of consequent issues:

a. What are the competencies we wish to assess?

b. What are the levels of these competencies, and how can we associate levels with these?

c. How do we assess, for instance, creativity, problem solving, proof skills etc.

d. How many times should we assess each competency?

e. How can we organise this style of assessment?

f. How will the students respond to a very different style of assessment?

g. What is the place of the examination, with a tight mark scheme in this view of the world?

h. What role should oral assessment play in any new scheme? Experience of colleagues from continental Europe suggests that we could do a lot more.

i. What role should electronic assessment play? There is currently a National HE STEM Programme funded project aiming to develop a user guide for people who wish to engage in electronic assessment. It might well be that we can get an approximate idea of the level of students with such a test, and then use other more delicate mechanisms to refine this view.

j. There should not be too much different assessment – because this is confusing for the students, but it also suggests that we do not quite know what we are doing.

In the new world we will need to ensure that any scheme that we have is defendable against legal challenge. I do not believe that the system we currently have is defensible against legal challenge. People in the arts have more subjectively described assessment schemes, such as the sort that I am suggesting above, without coming under attack. We just need to ensure that we are clear with students and give them a lot of feedback.

I believe that if we are clear enough about what we want to do with our students, QAA will be happy for us to do what we like. I think that the lack of a coordinated voice allows others to tell us what to do. Perhaps we will need to come to some compromise as to what we believe in order to get more control over what happens.

At this stage it might be instructive to try to deconstruct one area of activity, for instance proof, and describe a hierarchy of associated competencies. We can then think about how we might develop proof through a degree. This is for illustrative purposes and can surely be improved.

Third class:

- know by name a number of different methods of proof;
- and be able to prove a variety of simple theorems (which they must remember) demonstrating each method of proof;
- be able to recognise more straightforward errors in a more complex proof, for instance where logical quantifiers have been wrongly ordered, or where a division by zero may have happened;
- be able to use induction to prove a simple arithmetic identity.

2ii

- All of the above
- Be able to describe informally the proof of their own statement in words, but not necessarily codify with accurate mathematical formalism;
- To be able to use each of the main methods of proof in the above way.

2i

- All of the above;
- To be able to reproduce seen proofs using formal language;
- To be able to provide an outline of how to justify their own conjecture;
- To be able to go some way to a proof with more formal argumentation using appropriate mathematical language;
- To have facility all of the proof styles;

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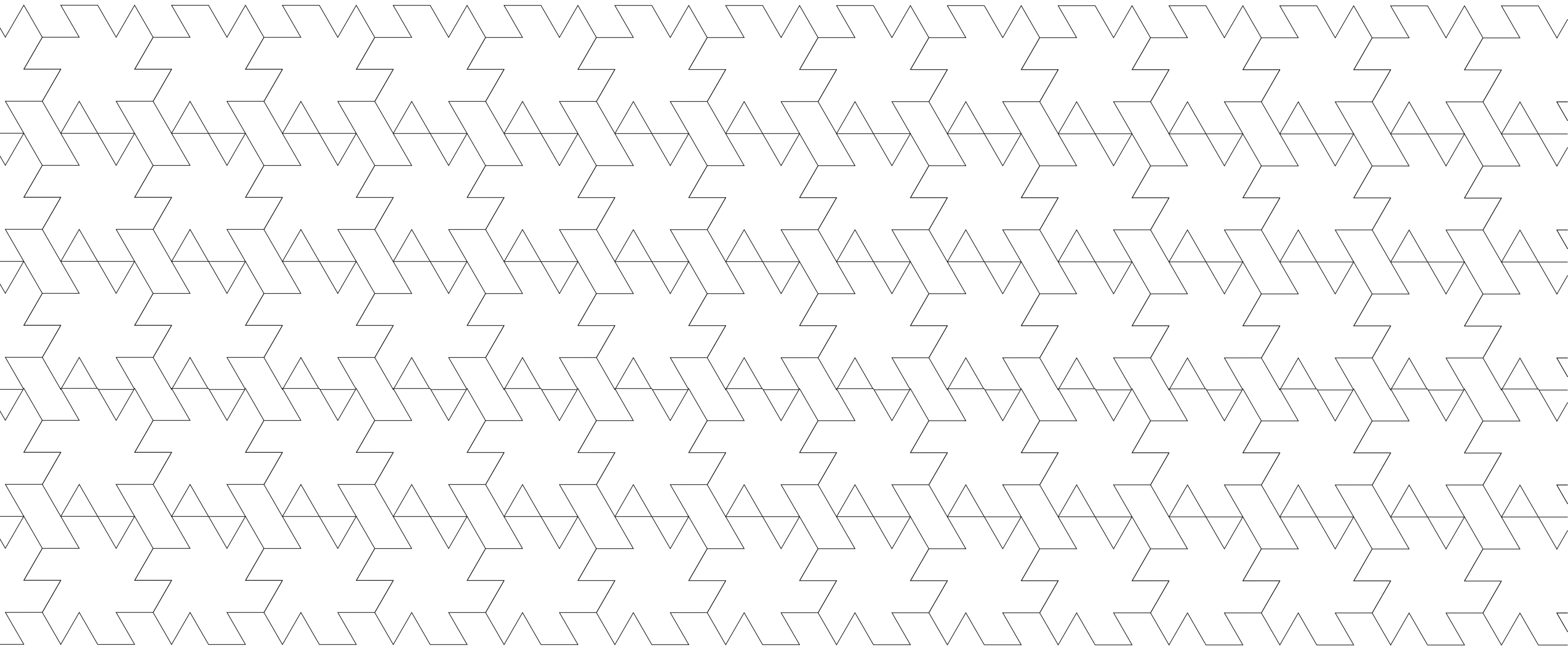
- All of the above;
- To be able to formally prove their own straightforward conjectures;

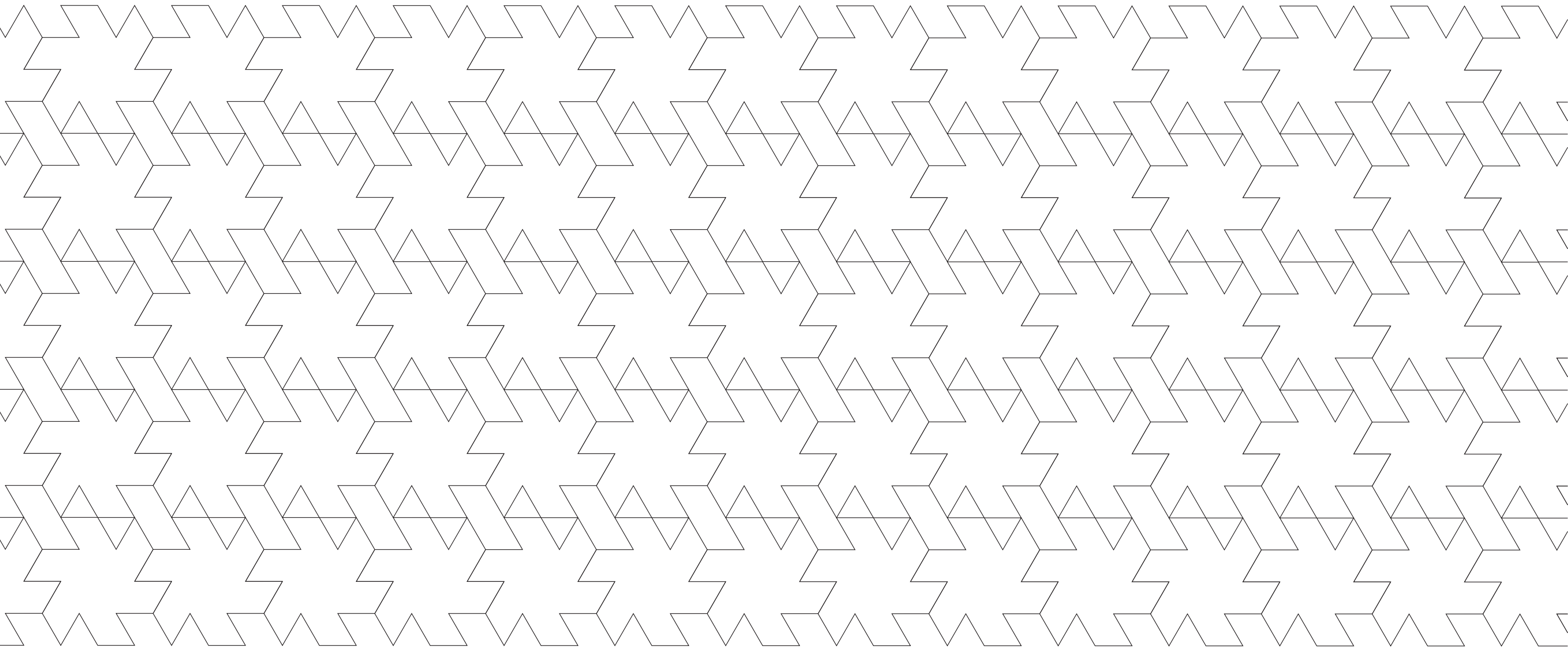
- To be able to read and understand more complex proofs of length a page or more.

I would then propose that we develop proof to a 2ii level in Year 1, to 2i level in Year 2 and only in Year 3 do we move onto first class proof activity. A student who does not progress beyond 2ii can continue to succeed at practising 2ii skills, and can aspire to the 2i skills, and so for 2i students and first class skills. The only information we need collect related to proof is that which tell us which level the student is at.

In conclusion, I suggest that a breakdown of a competency that we wish to develop into a hierarchy will lead to a much more efficient assessment regime, as we collect enough information to place the student against the set of descriptors. The feedback for the student is immediate, as they have a very concrete description of their level, and also a set of descriptors for how they might improve.

It is my conviction that we can, as a community, agree (more or less) on such a set of descriptors for each area of learning which we value. Following this we can develop an efficient scheme of assessment that will have national currency, and that we can argue to have accepted by our own institutions if we so wish.





5. 'What do the students think about their Maths degrees?'

by Dr. Neil Challis

(Based on joint work with Dr Mike Robinson and Dr Mike Thomlinson, More Maths Grads Project and Sheffield Hallam University)

The forthcoming sharp rise in tuition fees may mean students take much more interest in what they gain from their courses, and what they say is an important part of informing potential students about what is good and what is not.

One aspect of the HE Curriculum Theme of the recent More Maths Grads project was to glean information about both mathematics student and staff attitudes from a range of surveys and interviews in four diverse universities which included both research-intensive and post-92 institutions. Amongst the issues which arose across the patch were transition to university, graduate employability and career awareness, and comparisons of student and staff attitudes and aspirations.

When open questions were asked, human issues arose. It was apparent that amongst the students, there is a huge diversity of motivation, aspirations, background, experience, confidence, and preferred way of learning. The notion of everyone feeling they belong to a mathematical community is important, and there were interesting contrasts between student and staff responses.

Students admitted generally that their first place to go for help is their friends, so perhaps one role of academics is to ensure that a suitable community spirit exists. The nature of post-A level Mathematics came as "*a bit of a surprise*" to some, although nevertheless some found it "*more interesting and better*". However one well qualified student found it "*harder than what I expected*", and found some lecturers assumed a background of Further Maths even though it was not required for admission. As for teaching, some students found things "*strange*", "*totally different*", "*I just felt really stupid*".

We asked first year students why they chose to study mathematics. The most popular answers were enjoyment, being good at it and understanding that it can lead to wide job prospects. In fact 93% of respondents to one questionnaire rated getting a good job as an important or very important outcome of their degree studies. This varied little from one university to another. However when asked about career plans fewer than half had any idea. For those who did, the most commonly mentioned options were banking, finance, and accountancy, with teaching coming a little behind. Several mentioned "*earning a lot*" as a career plan! Specifically mathematical options such as "*research*" and "*something with maths*" lagged significantly behind.

Let us compare this with what arose from staff interviews across the universities. We find strong evidence of staff committed to good teaching, but they also have other priorities: "*Undergraduate teaching pays the rent. You do it well because you have more fun with it that way. But one large part of my motivation for teaching undergraduates [is] getting postgraduates doing research with me.*"

Staff realise that students are diverse in abilities and motivations, but much of what they say is focussed on perceived shortcomings: *"they don't understand fractions"; "algebraic manipulation, it's not second nature"; "schools seem to provide an algorithmic approach"; "A Level mentality is preparing for exams"; "coursework is for marks. They don't get a mark, they don't do it"; "students don't use mathematical language and they use equals signs with gay abandon"; "the first time they've ever met a proof, they have no idea.... what's expected"*.

In one telling quote, a lecturer goes straight to the heart of the contrast between staff and students: "*You know, we get all wrapped up in the syllabus and maths, definitions and proofs but, … our typical student ... wants a good 2(i) and a good job.*"

The preceding paragraphs provide a brief summary of some of the staff and student feelings we gleaned, - some might say almost a caricature, but notwithstanding that, there are some interesting perceptions that arise. We (the academics) clearly know our students are not like us. We cater for the needs of our students, but lose no opportunity to grumble about this, and to say, in effect, that we wish they were more like us. They are not. For most of them, their aspirations do not in general match our own aspirations either for ourselves or our students.

One can speculate about the inadvertent damage done by such a clash, through body language, throwaway remarks, everyday interaction. If your aspirations are not respected, student or staff, you can become disheartened and disillusioned, and carry negative messages back to the next generation of would-be maths graduates.

What does this have to do with curriculum? Curriculum design should be influenced by an awareness of the aspirations of those people who will study that curriculum. Some staff said the problems would be resolved by aiming not for more maths grads but for *better* maths grads via higher entry requirements. It is not clear though, that this would resolve the tension between staff and student aspirations, and what the effect would be overall on the UK mathematical ecosystem.

Perhaps a better route to follow is to note that the skills and attitudes that make a "good" mathematician are largely the same in nature as those valued by employers. Few of these skills are directly related to specific syllabus content. As one academic said:

> "*what people don't realise about a maths degree is that it's, it's a skills degree and when … you go off into work … you're not saying, 'Oh look I can integrate this.' It's to do with, 'Look I'm capable of thought on this level, I'm capable with this of dealing with this level of abstraction, I'm capable of using models and I'm capable of applying my brain in a really strange way, and problem solving.*"

Can we focus more explicitly on the skills and attitudes valued by both mathematicians and employers? We do need, of course, to create a coherent course, and to plan our topic areas as a vehicle to develop mathematical attitudes, but we are good at that already! If we have one flaw it is that we tend to overcrowd our syllabi. Thus we finish by posing this question: if we squeeze less material into our courses, would our students learn more, have more time to work on and develop good mathematical behaviours, become less strategic, become more satisfied with their course?

Points arising in discussion

A number of interesting issues arose during the post-talk discussion.

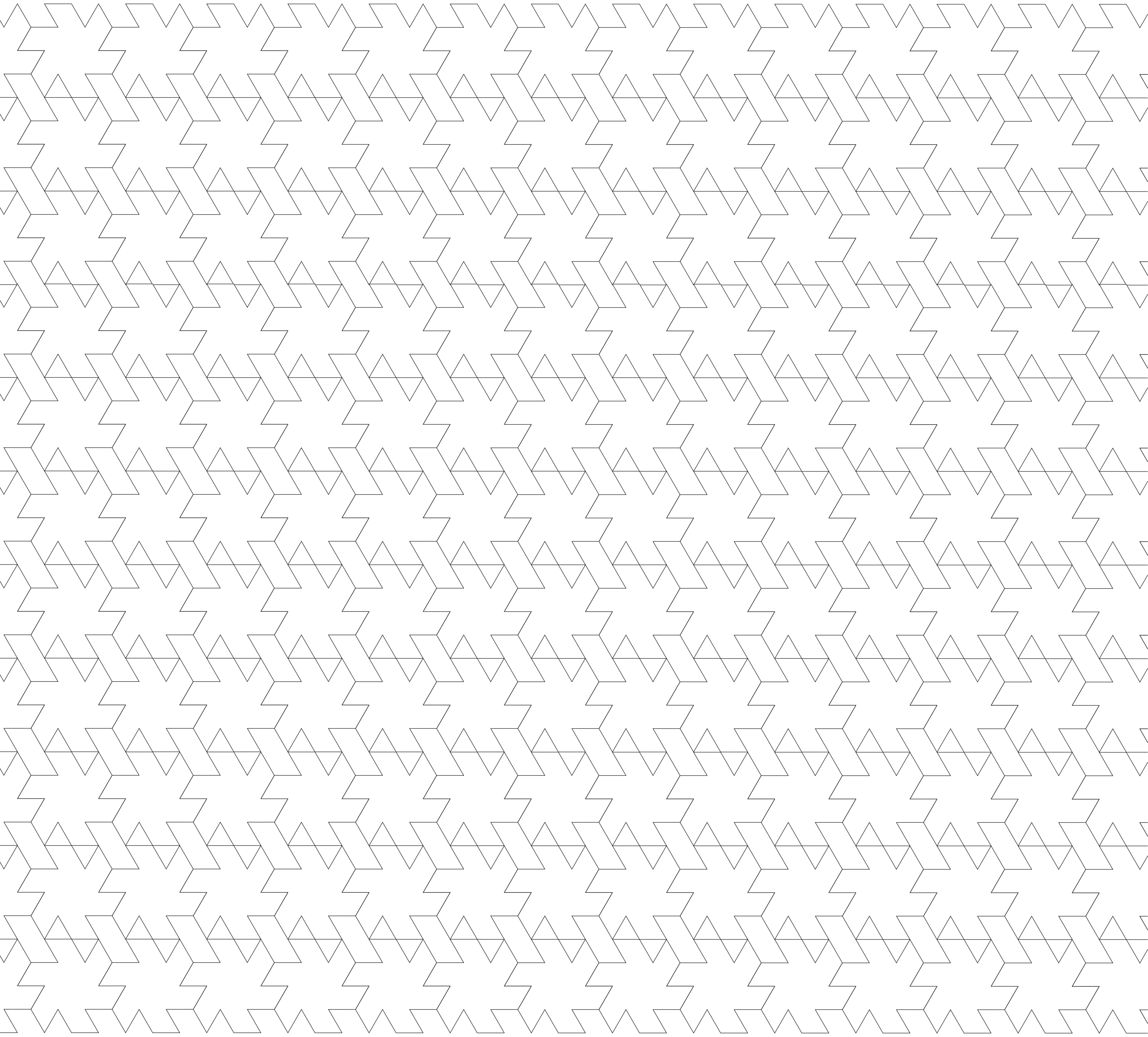
- Comment: "*We need to distinguish between skills and education and respect the cultural side of the subject too*" The speaker agreed and did not see any conflict here.
- Question: "*Did the research highlight any differences between research intensive and post-'92 universities?*" The answer is, surprisingly little. Perhaps staff at post-'92 establishments may be more explicit about employability, but students presented a remarkably even set of concerns about the kinds of job they may aspire to at the end of their course.
- Question: "*What did students think about themselves? Did they describe themselves as mathematicians?*" We did not explicitly ask that question - I wish we had.
- Question: "*You asked students why they chose maths. A key answer was enjoyment - can you elaborate on this?*" Some talked about enjoying being good at maths, but mostly when we discussed enjoyment, their answers turned to social life, and community and friendship groups which were based around friends on the same course.

Further reading

The findings of the More Maths Grads HE Curriculum theme are published in

Robinson M, Challis N and Thomlinson M (2010) *Maths at University: reflections on experience, practice and provision*

available online at http://maths.shu.ac.uk/moremathsgrads or on paper from the authors at n.challis@shu.ac.uk.



6. Recommendations for targeting financial support for curriculum development

The three discussion groups met again at the end of the Summit to consider, in the light of everything that had come up during the day, what were the priorities for funding projects in curriculum development. This section contains recommendations from the three groups for pieces of work that could be beneficial to undertake.

Problem solving

1. Sharing good practice: Collect case studies for how to embed problem solving into curricula. Develop a good practice guide for problem solving and assessment of problem solving. Consult existing sources, including George Pólya's 'How to Solve it'. Consider the questions: what is a problem and what makes a problem a useful teaching tool? Consider the teaching and assessment of unfamiliar problems and problems that are not easily solved, including lecturer and student confidence in approaching such problems. Consider the tension between rigorous proof and ways of approaching problems to get a 'useful' answer; does insistence on rigorous proof interfere with students' confidence in approaching unfamiliar problems?
2. Development of a bank of problems with solutions and extensions. Including unfamiliar problems, problems that are not easily solved, problems that have a correct answer but not a single best approach.
3. Development of a collection of teaching resources on the development of mathematics – stories from history and more recent development of the discipline. These should aim to counter a view of mathematics as a static, completed body of knowledge and instead encourage awareness of the process of doing mathematics. They should develop students' awareness of the culture of mathematics.

Industry

4. Development of a bank of industry-based problems. These are problems suitable for undergraduate students developed in consultation with industry partners and vetted. This would make industrial problems available to those without good industrial connections and avoid the resource intensive process of dealing with industry. Problems can be at different difficulty levels but should be vetted and rated so students are allocated problems of equivalent difficulty.
5. Pilot extending the model of the 'study groups with industry' to undergraduate project work.
6. Pilot of undergraduate students gaining experiencing of working in industry through short term placements (e.g. 2 hours per week).

Assessment

7. Research project to provide a review of existing theory of assessment schemes for mathematics and collect examples of good practice on use of different assessment methods for mathematics. Explore exemplars of innovative approaches to assessment. Consider assessment over the whole degree programme as well assessment at individual assignment or module level.
8. Sharing good practice: Develop a repository of assessment teaching resources. Develop a package of question design support for new lecturers.

Skills

9. Resource development: Development of maths-focused resources equivalent to already published generic resources on improving students' communication skills.

10. Building on the case studies collected by Mathematical Sciences HE Curriculum Innovation Project funded mini-project ‘Developing Graduate Skills in HE Mathematics Programmes’ led by Sheffield Hallam University.

Sustainability

11. Research how maths is addressing issues of sustainability (Education for Sustainable Development). Investigate links with the National HE STEM Programme funded ‘Green STEM’ project at University of Bradford.

Miscellaneous

12. Sharing good practice through an inter-university teacher exchange programme. This could be a reciprocal arrangement or one-way. For example, a lecturer may teach some classes or work in a maths support centre at another university. A lecturer may visit another university to observe and learn from some good practice, which could be brought back to the home university. Alternatively, a lecturer with some good practice to share might work in another university to establish use of that good practice there.

13. Fund undergraduate students to undertake focused summer intern projects within universities.

14. Research using social networking to find out about the destination of graduates and collect the feedback of graduates in employment on the mathematics HE curriculum.

The HE Mathematics Curriculum Summit on 12 January 2011 brought together representatives of half of the mathematical sciences departments in England and Wales and the professional bodies for a day of debate and discussion on the state of HE mathematics course design and delivery. This report gives summaries of the debate, talks and discussions as well as a series of recommendations of current priorities for curriculum development in mathematical sciences.

The Summit was funded by the Mathematical Sciences HE Curriculum Innovation Project, operated by the Maths, Stats and OR (MSOR) Network as part of the Mathematical Sciences Strand of the National HE STEM Programme. Find out more at **www.mathstore.ac.uk/hestem**