1. Scientific notation, powers and prefixes

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1.1 Rationale: why use scientific notation or powers?

In biology there are many instances where you might need to calculate and manipulate very large numbers or very small numbers. For example the number of nerve cells in an average brain might be 10000000000. On the other hand, the length of a cell under the microscope might be 0.000001m. The number of cell surface receptors for hormones might be 100000 per cell whilst the concentration of peptide hormone in the extracellular space might be 0.000000000001 M. These very large or very small numbers are difficult to read and that is why we use scientific notation or powers.

1.2 Writing very large numbers in scientific notation

Very large numbers can be rewritten as other numbers multiplied together. For example 100 is equal to 10 times 10 and we can write this as 10^2. The table shows how other larger numbers can be written.

10	= 10	= just one ten	= 10^1
100	= 10 x 10	= 2 tens multiplied together	= 10^2
1 000	= 10 x 10 x 10	= 3 tens multiplied together	= 10^3
10 000	= 10 x 10 x 10 x 10	= 4 tens multiplied together	= 10^4
100 000	= 10 x 10 x 10 x 10 x 10	= 5 tens multiplied together	= 10^5
10 000 000 000		= 10 tens multiplied together	= 10^{10}

Definition of terms:

Note that the terms “scientific notation”, “exponential notation”, “powers”, “exponents” all mean the same thing.

The numbers that you’re multiplying together are called the “**base**”. The number of times you multiply them together is called the “**power**” or “**exponent**”.
So in the last example,

10000 is written as “ten to the four” or 10^4,
10 is the **base** and 4 is the **power or exponent**.

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Some examples:

Example 1.1

Write 6000 in scientific notation…
This is just 6 x 1000 which is 6×10^3

Example 1.2

I have done an experiment to determine the concentration of drug in solution and the answer was 6237234 molecules/l. Write this in scientific notation.

Write 6.237234 and then count how many places you need to move the decimal point to the right …

In practice you would never be able to measure the concentration of drug to that degree of accuracy. Usually you would work out how many significant figures are appropriate in this instance.

You may decide to write it in 4 **significant figures** instead, 6.237×10^6.

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1.3 Writing very small numbers in scientific notation

We can use the same ideas when writing very small numbers.

1/10	= 0.1	$= 1/10^1$	$= 10^{-1}$
1/100	= 0.01	$= 1/10^2$	$= 10^{-2}$
1/1000	= 0.001	$= 1/10^3$	$= 10^{-3}$
1/10000	= 0.0001	$= 1/10^4$	$= 10^{-4}$

> there is a handy general rule to remember, $1 / 10^a = 10^{-a}$

Some examples:
Example 1.3 Write 0.00054 in scientific notation
Answer: 5.4×10^{-4}
This time you had to count how many places to move the decimal place to the left.

Example 1.4 Write 0.0134 in scientific notation
Answer: 1.34×10^{-2}

It is just a convention to put the decimal place after the first digit.
You could, if you wanted to, write this number in many different ways including:

0.134×10^{-1}
1.34×10^{-2}
13.4×10^{-3}

All you are doing is moving the decimal place and changing the power to compensate.

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1.4 Practice converting between normal numbers and scientific notation

It is important that you are familiar and confident with ***how to convert between normal numbers and scientific notation and vice versa***.

To write 6478 in scientific notation, write 6.478×10^3.
What you are doing is working out how many places to move the decimal point.

The expression "6.478×10^3" is just saying, "write 6.478 and move the decimal point three places to the right" giving 6478.

Or you can think of it as saying 6478 is the same as 6.478×1000 which is the same as 6.478×10^3

To write 0.00045 in scientific notation, write 4.5×10^{-4}
The expression "4.5×10^{-4}" is saying, "write 4.5 and move the decimal place four places to the left giving 0.00045."

Or you can think of it as saying $4.5 / 10^4$ or $4.5 / 10000$.

Some Examples:

Example 1.6
Write 340000 in scientific notation.
Answer: 3.4×10^5

Example 1.7
Write 0.0000080 in scientific notation.
Answer: 8×10^{-6}

Example 1.8 Fill in the gaps:
0.00475 can be written as _____ $\times 10^{-2}$ and ____ $\times 10^{-3}$ and ____ $\times 10^{-4}$
Answer: 0.0475×10^{-2} and 4.75×10^{-3} and 47.5×10^{-4}

Example 1.9
Write 9859486 in scientific notation to two significant figures
Answer: 9.9×10^6
(note that if the third digit is 5 or more, then the second digit is rounded up so in this case the third digit is 5 which means the second digit, 8, gets rounded up to 9.

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1.5 Add and subtract in scientific notation

To add or subtract two numbers in scientific notation,
you first need to convert them to the same power.

For example,
$5 \times 10^3 + 4 \times 10^5$
$= 5 \times 10^3 + 400 \times 10^3$
$= 405 \times 10^3$
$= 4.05 \times 10^5$

This is just the same as what you would normally do, i.e. you would line them up…

5000	5×10^3
+ 400000	400×10^3
= 405000	405×10^3

The same idea is used when subtracting,
$2 \times 10^{-3} - 8 \times 10^{-4}$
$= 20 \times 10^{-4} - 8 \times 10^{-4}$
$= 12 \times 10^{-4}$
$= 1.2 \times 10^{-3}$

This might be easier to visualise as…

0.0020	20×10^{-4}
- 0.0008	8×10^{-4}
= 0.0012	12×10^{-4}

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1.6 Multiply and divide in scientific notation

To multiply numbers with the same base, add the exponents

$a^b \times a^c = a^{b+c}$

Some examples:

Example 1.10 $10^3 \times 10^5 = 10^{3+5} = 10^8$

Example 1.11 $100 \times 10^3 = 10^2 \times 10^3 = 10^{2+3} = 10^5$

Here you have to convert 100 to 10^2 so you have the same base first before adding the powers.

Example 1.12 $6 \times 10^2 \times 5 \times 10^{10}$

Here you just multiply the 6 and 5 as you would normally do, then add the powers.

$=30 \times 10^{12}$

What is the power of a power? $(a^b)^c = a^{(b \times c)}$

Example 1.13 $(10^3)^3 = 10^3 \times 10^3 \times 10^3 = 10^{3 \times 3} = 10^9$

Example 1.14 $(10^{-5})^2 = 10^{-10}$

To divide numbers with the same base, subtract the exponents

$$\frac{a^b}{a^c} = a^{b-c}$$

Example 1.15 $\frac{10^9}{10^4} = 10^{9-4} = 10^5$

Example 1.16 $\frac{10^9}{10^{15}} = 10^{9-15} = 10^{-6}$

Example 1.17 $\frac{10^9}{10^{-15}} = 10^{9-(-15)} = 10^{9+15} = 10^{24}$

Example 1.18 $\frac{0.1}{10^3} = \frac{10^{-1}}{10^3} = 10^{-1-3} = 10^{-4}$

Here you have to convert 0.1 to 10^{-1} so you have the same base first before adding the powers.

Example 1.19 $\frac{5 \times 10^9}{2 \times 10^4} = 2.5 \times 10^{9-4} = 2.5 \times 10^5$

Here you just divide 5 by 2 as you would normally do, then subtract the powers.

But what if b – c gives zero?
If b - c is zero, then the exponents were the same and this is the same as dividing a number by itself which of course gives one.
$a^0 = 1$

Example 1.20 $\frac{10^5}{10^5} = 10^{5-5} = 10^0 = 1$

1.6b A note about fractional powers.

Once we understand that $10^a \times 10^b = 10^{a+b}$ then it becomes clear that the values for a and b do not need to be integers. For example, consider the following,

$$10^{0.5} \times 10^{0.5} = 10^{0.5+0.5} = 10^1 = 10$$

This is the same as writing: $10^{1/2} \times 10^{1/2} = 10^{1/2+1/2} = 10$

which is the same as: $\sqrt{10} \times \sqrt{10} = 10$

Similarly $10^{1/3} \times 10^{1/3} \times 10^{1/3} = 10$

is the same as writing: $\sqrt[3]{10} \times \sqrt[3]{10} \times \sqrt[3]{10} = 10$

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The symbols √ and $^3\surd$ are typically only used for square roots and cubed roots i.e. $10^{1/2}$ and $10^{1/3}$ respectively. Otherwise we just use the decimal notation eg $10^{0.793}$.

Fractional powers follow all the same rules as integer powers.

multiplying $10^a \times 10^b = 10^{a+b}$	example:	$10^{0.3} \times 10^{0.8} = 10^{1.1}$
dividing $\frac{10^a}{10^b} = 10^{a-b}$	example:	$\frac{10^{0.3}}{10^{0.9}} = 10^{0.3-0.9} = 10^{-0.6}$
powers of powers $(10^a)^b = 10^{a \times b}$	example:	$(10^{0.3})^{0.5} = 10^{0.15}$
For addition and subtraction we must convert to the same power, so:		$10^{-6.3} + 10^{-6.9}$ $= 5.01187 \times 10^{-7} + 1.2589 \times 10^{-7}$ $= 6.27077 \times 10^{-7}$

1.7 Prefixes

Prefixes are a useful way of abbreviating even further for example 10^{-3} g = 1 mg (one milligram)

Here is a summary of all of the standard prefixes. The main prefixes in use in biomedical science are shown in bold: learn them.

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	**10^{-1}**	**deci**	**d**
10^{21}	zetta	Z	**10^{-2}**	**centi**	**c**
10^{18}	exa	E	**10^{-3}**	**milli**	**m**
10^{15}	peta	P	**10^{-6}**	**micro**	**μ**
10^{12}	**tera**	**T**	**10^{-9}**	**nano**	**n**
10^{9}	**giga**	**G**	**10^{-12}**	**pico**	**p**
10^{6}	**mega**	**M**	**10^{-15}**	**femto**	**f**
10^{3}	**kilo**	**k**	**10^{-18}**	**atto**	**a**
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deca	da	10^{-24}	yocto	y

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1.8 Practice with prefixes

Some examples:

Example 1.21 Convert scientific notation to a prefix:
Convert 5×10^{-5} g to mg and then to μg.

5×10^{-5} g = $5 \times 10^{-2} \times 10^{-3}$g = 0.05 mg
or
= 50×10^{-6} g = 50 μg

Example 1.22 Convert a prefix to scientific notation:
12pmol = 12×10^{-12} mol = 1.2×10^{-11}mol

Example 1.23 Write 0.033 nM in scientific notation:
0.033nM = 0.033×10^{-9}M = 3.3×10^{-11}M

Example 1.24 Under the microscope, an epithelial cell looks quite rectangular and you can use the formula for the area of a rectangle to estimate the area of the cell. The dimensions you measure are width = 1 μm and length 10μm. Express the area in scientific notation with m^2 as the units.

Area = width x length = 1×10^{-6}m x 10×10^{-6}m
= $10 \times 10^{-12} m^2$

The height has been estimated from other studies to be approximately 5μm, what is the volume of the cell (in m^3 in scientific notation of the form $a \times 10^b$)?

Volume = area x height = $10 \times 10^{-12} m^2 \times 5 \times 10^{-6}$m
= 50×10^{-18} m^3
= 5×10^{-17} m^3

1.9 Supplementary material – SI Units

The standard units of measurement are the "SI Units" (Systeme Internationale) are given in the table below. A good website for reference is the National Physical Laboratory (www.npl.co.uk/reference/).

measurement	SI Unit	standard abbreviation
time	second	s
length	metre	m
mass	kilogram	kg
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

There are quite a few units which are in everyday use which are not in the above table. The important ones for biologists are the units for time, temperature and volume:

name	symbol	value in SI units
minute	min	1 min = 60 s
hour	h	1 h = 60 min = 3600 s
day	d	1 d = 24 h = 86 400 s
degrees Celsius	°C	temp in °C = (temp in K) – 273.15
litre	l, L	1 l = 1 dm^3 = 10^{-3} m^3

Note that the litre can be abbreviated as l or L but L is often used because of the potential for confusion of l ("ell") and 1 ("one").

Sometimes dm^3 is used instead of L and cm^3 is used instead of mL although L and mL are more common.

There are a few conventions and it is a good idea to follow them to avoid confusion. The key ones are:

- Unit symbols are unaltered in the plural (i.e. write 8 m not 8 ms to mean 8 metres.)
- Abbreviations such as sec (for either s or second) or mps (for either m/s or meter per second) are not allowed.
- A space is left between the numerical value and unit symbol (25 kg but not: 25-kg or 25kg).
- Mathematical operations should only be applied to unit symbols (kg/m^2) and not unit names (kilogram/cubic metre).
- kg/m^2 can also be written as $kg.m^{-2}$

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1.10 Converting between units for volume

Volume is sometimes expressed in dm^3, or cm^3 or litres or ml – it is essential to be able to convert between them.

Converting volumes between m^3 and L.

A spherical bacterial cell has a diameter of approximately 4 μm, what would its volume be (in m^3 and in L)?

There are (at least) two possible approaches to this.

Method 1: The easiest way to do this is to change the units of the dimensions to dm first.

The volume of a sphere is
$4/3\ \pi r^3$.
The radius is 2 μm = 2×10^{-6} m = 2×10^{-5} dm

Volume = $4/3 \times 3.14 \times (2 \times 10^{-5}\ \text{dm})^3$
= $33.5 \times 10^{-15}\ \text{dm}^3$
= 33.5 fL

Method 2: An alternative method is to calculate the volume in μm^3 and convert the answer from μm^3 directly to L.

Volume = $4/3 \times 3.14 \times (2\ \mu\text{m})^3 = 33.5\ (\mu\text{m})^3$
I have included the bracket around the "μm" here to point out that it is a "micrometer cubed" that is, $(10^{-6}\text{m})^3$ which is 10^{-18}m^3. This is not the same thing as $\mu(\text{m})^3$ which would be 10^{-6}m^3. Commonly when you see μm^3 written down it means $(\mu m)^3$.

Volume = $33.5 \times (10^{-6})^3 = 33.5 \times 10^{-18}\ \text{m}^3$

So now we need to convert $33.5 \times 10^{-18}\text{m}^3$ to litres.

One way to do this is to say,
$33.5 \times 10^{-18}\text{m}^3 \times (10\ \text{dm.m}^{-1})^3 \times 1\text{L.dm}^{-3}$
translating this into words… we take $33.5 \times 10^{-18}\text{m}^3$ and multiply by 10 decimeters per m three times and by one litre per decimetre cubed. You can see that the units cancel out to leave us with..

$33.5 \times 10^{-18} \times (10)^3$ L

So we are left with 33.5×10^{-15} L = 33.5 fL

Summary of Learning Outcomes

At the end of this section, you should be able to do the following confidently and reliably.

1. Convert between normal numbers and scientific notation and vice versa
2. Use the basic rules of manipulating powers and scientific notation, i.e. add, subtract, multiply and divide.
3. To know the common prefixes.
4. To be able to use these prefixes in calculations.
5. Recognise SI Units,
6. Convert between units for volume $dm^3 \leftrightarrow L$

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