Conservation of Momentum

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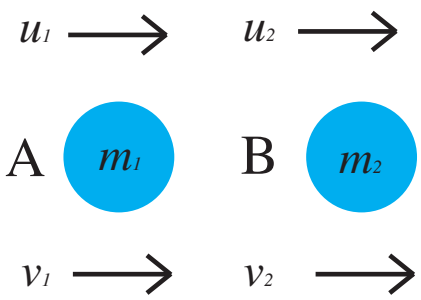
In this leaflet the concept of conservation of momentum will be introduced in both one and two dimensions.

Conservation of momentum

When no resultant external force acts on a system of interacting particles the total momentum of the system remains constant. Suppose two particles collide.



Given,

m_1 = mass of particle A
m_2 = mass of particle B
u_1 = velocity of particle A before collision
u_2 = velocity of particle B before collision
v_1 = velocity of particle A after collision
v_2 = velocity of particle B after collision

Then, the principle of conservation of momentum is:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

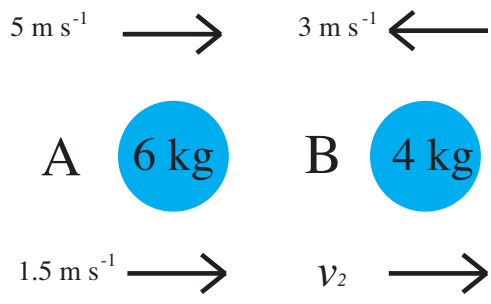
Total momentum before collision = Total momentum after collision

Worked Example 1.

A particle A, of mass 6 kg, travelling in a straight line at 5 m s^{-1} collides with a particle B, of mass 4 kg, travelling in the same straight line, but in the opposite direction, with a speed of 3 m s^{-1}. Given that after the collision particle A continues to move in the same direction at 1.5 m s^{-1}, what speed does particle B move with after the collision?

Solution

It is always useful to depict the collision with the velocities both before and after.



Using the principle of conservation of momentum:

$$\begin{aligned} m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\ 6 \times 5 + 4 \times (-3) &= 6 \times 1.5 + 4 \times v_2 \\ 9 &= 4 \times v_2 \\ v_2 &= \tfrac{9}{4} = 2.3 \text{ m s}^{-1} \quad (2 \text{ s.f.}) \end{aligned}$$

So after the collision particle B moves with a speed of 2.3 m s^{-1}, in the same direction as A.

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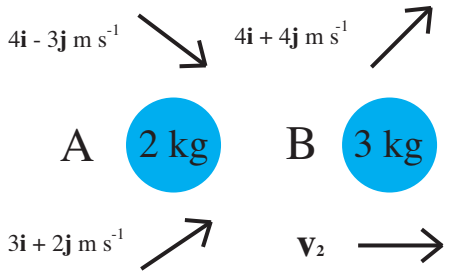
Conservation of momentum in two dimensions

The principle of conservation of momentum can be used in two dimensions by expressing the velocities in vector form to obtain: $m_1\mathbf{u_1} + m_2\mathbf{u_2} = m_1\mathbf{v_1} + m_2\mathbf{v_2}$.

Worked Example 2.
A particle A, of mass 2 kg, collides with a particle B, of mass 3 kg. The velocity of particle A before the collision was $(4\mathbf{i} - 3\mathbf{j})$ m s^{-1} and the velocity of particle B before the collision was $(4\mathbf{i} + 4\mathbf{j})$ m s^{-1}. Given the velocity of particle A after the collision was $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1}, what was the velocity of B after the collision?

Solution
Here we have, $m_1 = 2$, $m_2 = 3$, $\mathbf{u_1} = (4\mathbf{i} - 3\mathbf{j})$, $\mathbf{u_2} = (4\mathbf{i} + 4\mathbf{j})$ and $\mathbf{v_1} = (3\mathbf{i} + 2\mathbf{j})$

$$
\begin{aligned}
m_1\mathbf{u_1} + m_2\mathbf{u_2} &= m_1\mathbf{v_1} + m_2\mathbf{v_2} \\
2(4\mathbf{i} - 3\mathbf{j}) + 3(4\mathbf{i} + 4\mathbf{j}) &= 2(3\mathbf{i} + 2\mathbf{j}) + 3\mathbf{v_2} \\
8\mathbf{i} - 6\mathbf{j} + 12\mathbf{i} + 12\mathbf{j} &= 6\mathbf{i} + 4\mathbf{j} + 3\mathbf{v_2} \\
14\mathbf{i} + 2\mathbf{j} &= 3\mathbf{v_2} \\
\mathbf{v_2} &= \tfrac{14}{3}\mathbf{i} + \tfrac{2}{3}\mathbf{j} \\
&= 4.7\mathbf{i} + 0.67\mathbf{j} \text{ m s}^{-1} \text{ (2 s.f.)}
\end{aligned}
$$



Exercises

1. A particle A, of mass 10 kg, travelling in a straight line at 4 m s^{-1} collides with a stationary particle B, of mass 12 kg. Given that after the collision particle A becomes stationary, what speed does particle B move with?

2. A particle A, of mass m_1 kg, travelling in a straight line at 6 m s^{-1} collides with a particle B, of mass 9 kg, which is travelling in the same direction, with a speed of 2 m s^{-1}. Given that after the collision particle A continues to move in the same direction at 1.5 m s^{-1} and particle B moves in the same direction as particle A at 5 m s^{-1}, what is the mass of particle A?

3. A particle A, of mass 2 kg, collides with a particle B, of mass 2 kg. The velocity of particle A before the collision was $(5\mathbf{i} + 4\mathbf{j})$ m s^{-1} and the velocity of particle B before the collision was $7\mathbf{i}$ m s^{-1}. Given the velocity of particle B after the collision was $(6\mathbf{i} + 4\mathbf{j})$ m s^{-1}, what was the velocity of particle A after the collision?

4. A particle A, of mass 8 kg, collides with a particle B, of mass m_2 kg. The velocity of particle A before the collision was $(-1\mathbf{i} + 4\mathbf{j})$ m s^{-1} and the velocity of particle B before the collision was $(-0.8\mathbf{i} + 1.4\mathbf{j})$ m s^{-1}. Given the velocity of particle A after the collision was $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1}, and the velocity of particle B was $3\mathbf{j}$ m s^{-1}, what was the mass of particle B?

Answers (All to 2 s.f.)

1. 3.3 m s^{-1}
2. 6 kg
3. $6\mathbf{i}$ m s^{-1}
4. 10 kg

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