The slope-intercept form

Introduction

One form of the equation of a straight line is called the **slope-intercept** form because it contains information about these two properties.

The equation of a straight line

Any equation of the form

\[ y = mx + c \]

where \( m \) and \( c \) are fixed numbers, (i.e. constants), has a graph which is a straight line.

For example,

\[ y = 3x + 5, \quad y = \frac{2}{3}x + 8 \quad \text{and} \quad y = -3x - 7 \]

all have graphs which are straight lines.

The slope and intercept of a straight line

In the equation \( y = mx + c \) the value of \( m \) is called the **slope**, (or gradient), of the line. It can be positive, negative or zero. Lines with a positive gradient slope upwards, from left to right. Lines with a negative gradient slope downwards from left to right. Lines with a zero gradient are horizontal.

The value of \( c \) is called the **vertical intercept** of the line. It is the value of \( y \) when \( x = 0 \). When drawing a line, \( c \) gives the position where the line cuts the vertical axis.
Example

Determine the gradient and vertical intercept of each line.

a) \( y = 12x - 6 \),  
    b) \( y = 5 - 2x \),  
    c) \( 4x - y + 13 = 0 \),  
    d) \( y = 8 \),  
    e) \( y = 4x \).

Solution

a) Comparing \( y = 12x - 6 \) with \( y = mx + c \) we see that \( m = 12 \), so the gradient of the line is 12. The fact that this is positive means that the line slopes upwards as we move from left to right. The vertical intercept is \(-6\). This line cuts the vertical axis below the horizontal axis.

b) Comparing \( y = 5 - 2x \) with \( y = mx + c \) we see that \( m = -2 \), so the gradient is \(-2\). The line slopes downwards as we move from left to right. The vertical intercept is 5.

c) We write \( 4x - y + 13 = 0 \) in standard form as \( y = 4x + 13 \) and note that \( m = 4, c = 13 \).

d) Comparing \( y = 8 \) with \( y = mx + c \) we see that \( m = 0 \) and \( c = 8 \). This line is horizontal.

e) Comparing \( y = 4x \) with \( y = mx + c \) we see that \( m = 4 \) and \( c = 0 \).

Exercises

1. State the gradient and intercept of each of the following lines.
   
a) \( y = 5x + 6 \),  
b) \( y = 3x - 11 \),  
c) \( y = -2x + 7 \),  
d) \( y = 9 \),  
e) \( y = 7 - x \)

Answers

1. a) gradient 5, intercept 6  
    b) 3,−11  
    c) −2,7  
    d) 0,9  
    e) −1, 7.

More about the gradient

The gradient measures the steepness of the line. A large positive value of \( m \) means the graph increases steeply as you move from the left to the right. A small, but positive value of \( m \) means the graph increases, but not very steeply. Similarly, a large negative value of \( m \) means that the graph drops steeply as you move from left to right. A small negative value means the graph decreases, but not very steeply.

In fact we can say more. The value of \( m \) tells us the amount by which \( y \) increases (or decreases) if \( x \) increases by one unit.

For example, for the line \( y = 5x + 13 \), the value of \( y \) increases by 5 units every time \( x \) increases by 1 unit.

In the line \( y = -3x + 7 \) the value of \( y \) decreases by 3 units every time \( x \) increases by 1 unit. You should sketch these graphs to convince yourself of this behaviour.

Exercises

1. If \( P = 4Q + 9 \), by what amount will \( P \) increase if \( Q \) increases by 1 unit ?
2. If \( P = 11 - 3Q \), by what amount will \( P \) decrease if \( Q \) increases by 1 unit ?
3. If \( P = 19 \), by what amount will \( P \) increase if \( Q \) increases by 1 unit ?

Answers

1. 4.  
2. 3.

3. It will not. The value of \( P \) is constant, that is fixed at 19. It does not depend upon \( Q \).