

# Integration

$f(x)$	$\int f(x) dx = F(x) + c$	
$k$ , constant	$kx + c$	
$x^n$ , ( $n \neq -1$ )	$\frac{x^{n+1}}{n+1} + c$	
$x^{-1} = \frac{1}{x}$	$\begin{cases} \ln x + c & x > 0 \\ \ln(-x) + c & x < 0 \end{cases}$	
$e^x$	$e^x + c$	
$\cos x$	$\sin x + c$	
$\sin x$	$-\cos x + c$	
$\tan x$	$\ln(\sec x) + c$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
$\sec x$	$\ln(\sec x + \tan x) + c$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
$\operatorname{cosec} x$	$\ln(\operatorname{cosec} x - \cot x) + c$	$0 < x < \pi$
$\cot x$	$\ln(\sin x) + c$	$0 < x < \pi$
$\cosh x$	$\sinh x + c$	
$\sinh x$	$\cosh x + c$	
$\tanh x$	$\ln \cosh x + c$	
$\coth x$	$\ln \sinh x + c$	$x > 0$
$\frac{1}{x^2 + a^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a} + c$	$a > 0$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a} \ln \frac{x-a}{x+a} + c$	$ x  > a > 0$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a} \ln \frac{a+x}{a-x} + c$	$ x  < a$
$\frac{1}{\sqrt{x^2 + a^2}}$	$\sinh^{-1} \frac{x}{a} + c$	$a > 0$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1} \frac{x}{a} + c$	$x \geq a > 0$
$\frac{1}{\sqrt{x^2 + k}}$	$\ln(x + \sqrt{x^2 + k}) + c$	
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1} \frac{x}{a} + c$	$-a \leq x \leq a$
$f(ax + b)$	$\frac{1}{a} F(ax + b) + c$	$a \neq 0$
e.g. $\cos(2x - 3)$	$\frac{1}{2} \sin(2x - 3) + c$	

## The linearity rule for integration

$$\int (af(x) + bg(x)) \, dx = a \int f(x) \, dx + b \int g(x) \, dx,$$

( $a, b$  constant)

## Integration by substitution

$$\int f(u) \frac{du}{dx} dx = \int f(u) du \quad \text{and}$$

$$\int_a^b f(u) \frac{du}{dx} dx = \int_{u(a)}^{u(b)} f(u) du$$

## Integration by parts

$$\int_a^b u \frac{dv}{dx} dx = [uv]_a^b - \int_a^b \frac{du}{dx} v \, dx$$

## Alternative form:

$$\int_a^b f(x)g(x) \, dx = \left[ f(x) \int g(x) dx \right]_a^b - \int_a^b \frac{df}{dx} \left\{ \int g(x) dx \right\} dx$$