Module 4.

Understanding and using logs and exponential equations

4J Logs to base 2 – plotting logs with different bases

Although, until now, we have talked only about logs to the base 10 and scientific notation as 10^a, it is of course possible to use any base. However, the only other bases commonly used in biology are base 2 and base e. More about e later, let's look at base 2.

Bacterial growth. An example of base 2.

Say you start off (time = t = 0) with a flask containing a certain number of bacteria N_0.
If the generation time is T (the period between one cell division and the next) and if each cell divides into two at the end of one generation time, then...

At T (time) =	N (number of bacteria) =	if N0 = 1 N=	$\log_2 N$
0	$N = N_0$	$1 = 2^0$	0
1	$N = 2N_0$	$2 = 2^1$	1
2	$N = 4N_0$	$4 = 2^2$	2
3	$N = 8N_0$	$8 = 2^3$	3
4	$N = 16N_0$	$16 = 2^4$	4
5	$N = 32N_0$	$32 = 2^5$	5
T	$N = 2^T N_0$	2^T	T

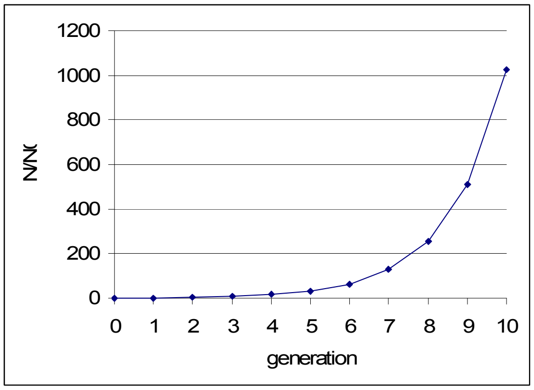
$$N = N_0 2^T \quad \leftrightarrow \quad \log N = T\log 2 + \log N_0$$

Now this is true for any base, but if we happen to choose base 2, then $\log_2 2 = 1$

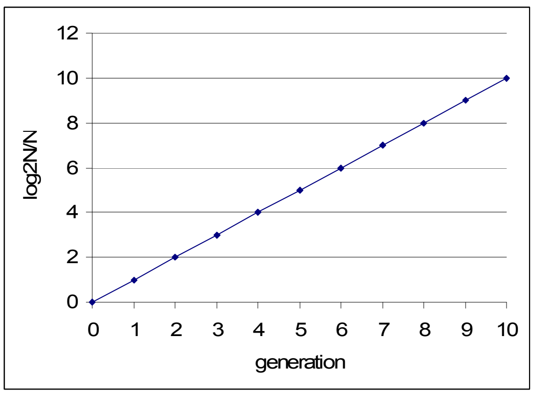
$$\log N = T + \log N_0$$

This is an equation for a straight line where $y = mx + c$ (m = 1)

Now take some data for N, plot this as N/N_0 vs generation number



Then plot it as $\log_2 N/N_0$ vs generation number and you get a straight line. Why?



In the log plot it is much easier to see that the population size is doubling with each generation and that the growth is exponential.

In practice this only occurs if there are plenty of nutrients and the bacteria don't produce anything that might limit their growth. The phase of growth where the generation time is constant is known as the exponential growth phase or the logarithmic growth phase.

Most calculators don't have a button for $\log_2 x$. But they do have a button for $\log_{10}$. So now calculate and plot log10

At T (time) =	N (number of bacteria) =	if N0 = 1 N=	$\log_{10} N$
0	$N = N_0$	1	0
1	$N = 2N_0$	2	0.3
2	$N = 4N_0$	4	0.6
3	$N = 8N_0$	8	0.9
4	$N = 16N_0$	16	1.2
5	$N = 32N_0$	32	1.5
T	$N = 2^T N_0$	2^T	

Now the equation is $N = N_0 2^T \leftrightarrow$

$$\begin{aligned} \log_{10}N &= T\log_{10}2 + \log_{10}N_0 \\ \log_{10}N &= 0.3T + \log_{10}N_0 \\ y &= mx + c \end{aligned}$$

So this is a straight line with slope = 0.3 and intercept $\log_{10}N_0$

4K Changing base

Say you want to work out $\log_2 60$ but your calculator only has the ability to calculate $\log_{10}$ and ln. What can you do? You can change the base of a logarithm…

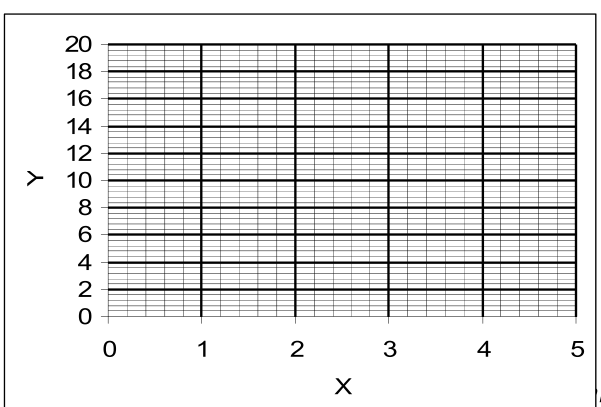
$\log_2 a = b \leftrightarrow 2^b = a$

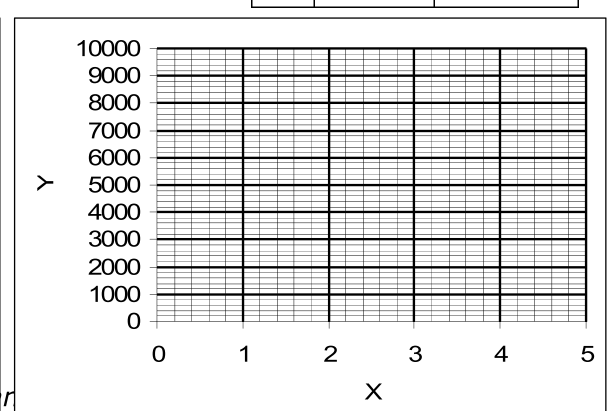
now take $2^b = a$ and take logs (to the base 10) of both sides
$\log_{10}(2^b) = \log a$
$b\log_{10}2 = \log a$
but $b = \log_2 a$
so $\log_2 a \times \log_{10}2 = \log a$
so $\log_2 a = \dfrac{\log_{10} a}{\log_{10} 2} = \dfrac{\log_{10} a}{0.301}$ or more generally, $\log_a x = \dfrac{\log_b x}{\log_b a}$
so $\log_2 60 = \dfrac{\log_{10} 60}{\log_{10} 2} = \dfrac{1.778}{0.301} = 5.9$

4L Graphing exponential growth and decay

All equations of the form $y = a^x$ have the same sort of shape.
If you are ever confronted with an equation which looks unfamiliar – try to sketch it to get a sense of what it looks like.

x	$y = 2^x$	$y = 10^x$
0	1	1
1	2	10
2	4	100
3	8	1000
4	16	10000





What happens if the power is negative? What does $y = a^{-x}$ look like?

x	$y = 2^{-x}$	$y = 10^{-x}$
0	1	1
1	1/2 = 0.5	1/10 = 0.1
2	¼ = 0.25	1/100 = 0.01
3	1/8 = 0.125	1/1000 = 0.001
4	1/16 = 0.0625	1/10000 = 0.0001