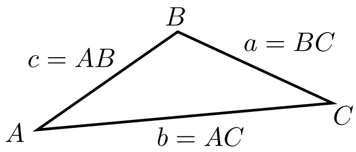
The sine rule and cosine rule

Introduction

To **solve** a triangle is to find the lengths of each of its sides and all its angles. The **sine rule** is used when we are given either a) two angles and one side, or b) two sides and a non-included angle. The **cosine rule** is used when we are given either a) three sides or b) two sides and the included angle.

1. The sine rule

Study the triangle ABC shown below. Let B stands for the angle at B. Let C stand for the angle at C and so on. Also, let $b = AC$, $a = BC$ and $c = AB$.



The sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example
In triangle ABC, $B = 21°$, $C = 46°$ and $AB = 9$cm. Solve this triangle.

Solution
We are given two angles and one side and so the sine rule can be used. Furthermore, since the angles in any triangle must add up to 180° then angle A must be 113°. We know that $c = AB = 9$. Using the sine rule

$$\frac{a}{\sin 113°} = \frac{b}{\sin 21°} = \frac{9}{\sin 46°}$$

So,

$$\frac{b}{\sin 21°} = \frac{9}{\sin 46°}$$

from which

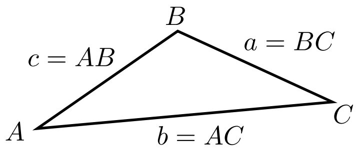
$$b = \sin 21° \times \frac{9}{\sin 46°} = 4.484\text{cm.} \qquad \text{(3dp)}$$

Similarly

$$a = \sin 113° \times \frac{9}{\sin 46°} = 11.517\text{cm.} \qquad \text{(3dp)}$$

2. The cosine rule

Refer to the triangle shown below.



The cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A, \qquad b^2 = a^2 + c^2 - 2ac\cos B, \qquad c^2 = a^2 + b^2 - 2ab\cos C$$

Example
In triangle ABC, $AB = 42$cm, $BC = 37$cm and $AC = 26$cm. Solve this triangle.

Solution
We are given three sides of the triangle and so the cosine rule can be used. Writing $a = 37$, $b = 26$ and $c = 42$ we have

$$a^2 = b^2 + c^2 - 2bc\cos A$$

from which

$$37^2 = 26^2 + 42^2 - 2(26)(42)\cos A$$

$$\cos A = \frac{26^2 + 42^2 - 37^2}{(2)(26)(42)} = \frac{1071}{2184} = 0.4904$$

and so

$$A = \cos^{-1} 0.4904 = 60.63°$$

You should apply the same technique to verify that $B = 37.76°$ and $C = 81.61°$. You should also check that the angles you obtain add up to 180°.

Exercises
1. Solve the triangle ABC in which $AC = 105$cm, $AB = 76$cm and $A = 29°$.

2. Solve the triangle ABC given $C = 40°$, $b = 23$cm and $c = 19$cm.

Answers
1. $a = 53.31$cm, $B = 107.28°$, $C = 43.72°$.   2. $A = 11.09°$, $B = 128.91°$, $a = 5.69$cm.