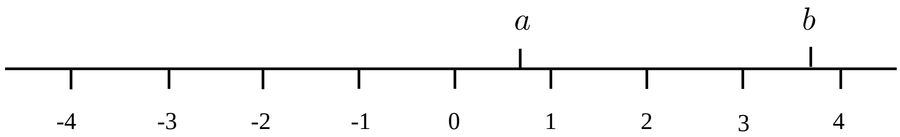
Inequalities

Introduction

The inequality symbols $<$ and $>$ arise frequently in engineering mathematics. This leaflet revises their meaning and shows how expressions involving them are manipulated.

1. The number line and inequality symbols

A useful way of picturing numbers is to use a **number line**. The figure shows part of this line. Positive numbers are on the right-hand side of this line; negative numbers are on the left.



Numbers can be represented on a number line. If $a < b$ then equivalently, $b > a$.

The symbol $>$ means 'greater than'; for example, since 6 is greater than 4 we can write $6 > 4$. Given any number, all numbers to the right of it on the line are greater than the given number. The symbol $<$ means 'less than'; for example, because -3 is less than 19 we can write $-3 < 19$. Given any number, all numbers to the left of it on the line are less than the given number.

For any numbers a and b, note that if a is less than b, then b is greater than a. So the following two statements are equivalent: $a < b$ and $b > a$. So, for example, we can write $4 < 17$ in the equivalent form $17 > 4$.

If $a < b$ and $b < c$ we can write this concisely as $a < b < c$. Similarly if a and b are both positive, with b greater than a we can write $0 < a < b$.

2. Rules for manipulating inequalities

To change or rearrange statements involving inequalities the following rules should be followed:

Rule 1. Adding or subtracting the same quantity from both sides of an inequality leaves the inequality symbol unchanged.

Rule 2. Multiplying or dividing both sides by a **positive** number leaves the inequality symbol unchanged.

Rule 3. Multiplying or dividing both sides by a **negative** number **reverses the inequality**. This means $<$ changes to $>$, and vice versa.

So,

$$\text{if } a < b \quad \text{then} \quad a + c < b + c \qquad \text{using Rule 1}$$

For example, given that $5 < 7$, we could add 3 to both sides to obtain $8 < 10$ which is still true.

Also, using Rule 2,

$$\text{if} \quad a < b \quad \text{and} \quad k \quad \text{is positive, then} \quad ka < kb$$

For example, given that $5 < 8$ we can multiply both sides by 6 to obtain $30 < 48$ which is still true.

Using Rule 3

$$\text{if } a < b \text{ and } k \text{ is negative, then} \quad ka > kb$$

For example, given $5 < 8$ we can multiply both sides by -6 and reverse the inequality to obtain $-30 > -48$, which is a true statement. A common mistake is to forget to reverse the inequality when multiplying or dividing by negative numbers.

3. Solving inequalities

An inequality will often contain an unknown variable, x, say. To **solve** means to find all values of x for which the inequality is true. Usually the answer will be a range of values of x.

Example
Solve the inequality $7x - 2 > 0$.

Solution
We make use of the Rules to obtain x on its own. Adding 2 to both sides gives

$$7x > 2$$

Dividing both sides by the positive number 7 gives

$$x > \frac{2}{7}$$

Hence all values of x greater than $\frac{2}{7}$ satisfy $7x - 2 > 0$.

Example
Find the range of values of x satisfying $x - 3 < 2x + 5$.

Solution
There are many ways of arriving at the correct answer. For example, adding 3 to both sides:

$$x < 2x + 8$$

Subtracting $2x$ from both sides gives

$$-x < 8$$

Multiplying both sides by -1 and **reversing the inequality** gives $x > -8$. Hence all values of x greater than -8 satisfy $x - 3 < 2x + 5$.

Exercises
In each case solve the given inequality.

1. $2x > 9$, 2. $x + 5 > 13$, 3. $-3x < 4$, 4. $7x + 11 > 2x + 5$, 5. $2(x + 3) < x + 1$

Answers
1. $x > 9/2$, 2. $x > 8$, 3. $x > -4/3$, 4. $x > -6/5$, 5. $x < -5$.

