The graph of a function

Introduction

A very useful pictorial representation of a function is the graph. On this leaflet we remind you of important conventions when graph plotting.

1. The graph of a function

Consider the function \( f(x) = 5x + 4 \).

We can choose several values for the input and calculate the corresponding outputs. We have done this for integer values of \( x \) between \(-3\) and \(3\) and the results are shown in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>(-11)</td>
<td>(-6)</td>
<td>(-1)</td>
<td>(4)</td>
<td>(9)</td>
<td>(14)</td>
<td>(19)</td>
</tr>
</tbody>
</table>

To plot the graph we first draw a pair of axes - a vertical axis and a horizontal axis. These are drawn at right-angles to each other and intersect at the origin \( O \) as shown below.

Each pair of input and output values can be represented on a graph by a single point. The input values are measured along the horizontal axis and the output values along the vertical axis. A uniform scale is drawn on each axis sufficient to accommodate all the required points. The points plotted in this way are then joined together, in this case by a straight line. This is the graph of the function. Each point on the graph can be represented by a pair of coordinates in the form \((x, f(x))\). Each axis should be labelled to show its variable.

2. Dependent and independent variables

The horizontal axis is often called the \( x \) axis. The vertical axis is commonly referred to as the \( y \) axis. So, we often write the function above, not as \( f(x) = 5x + 4 \), but rather as

\[
y = 5x + 4
\]
Since \( x \) and \( y \) can have a number of different values they are variables. Here \( x \) is called the **independent variable** and \( y \) is called the **dependent variable**. Knowing or choosing a value of the independent variable, \( x \), the function allows us to calculate the corresponding value of the dependent variable, \( y \). To show this dependence we often write \( y(x) \). This notation simply means that \( y \) depends upon \( x \). Note that it is the independent variable which is the input to the function and the dependent variable which is the output.

**Example**
Consider the function given by \( y = 2t^2 + 1 \), for values of \( t \) between \(-2\) and \(2\).

a) State the independent variable.

b) State the dependent variable.

c) Plot a graph of the function.

**Solution**
a) The independent variable is \( t \).

b) The dependent variable is \( y \).

c) A table of input and output values should be constructed first. Such a table is shown below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Each pair of \( t \) and \( y \) values in the table is plotted as a single point. The points are then joined with a smooth curve to produce the required graph as shown below.

**Exercises**
1. Plot a graph of each of the following functions. In each case state the dependent and independent variables.

   a) \( y = f(x) = 3x + 2 \), for \( x \) between \(-2\) and \(5\),

   b) \( y = f(t) = 6 - t^2 \), for \( t \) between \(1\) and \(5\).

**Answers**
1. a) dependent variable is \( y \), independent variable is \( x \).

   b) dependent variable is \( y \), independent variable is \( t \).