What is a surd?

Introduction

In engineering calculations, numbers are often given in surd form. This leaflet explains what is meant by surd form, and gives some circumstances in which surd forms arise.

1. Surd form

Suppose we wish to simplify \( \sqrt{\frac{1}{4}} \). We can write it as \( \frac{1}{2} \). On the other hand, some numbers involving roots, such as \( \sqrt{2}, \sqrt{3}, \sqrt{6} \) cannot be expressed exactly in the form of a fraction. Any number of the form \( \sqrt[n]{a} \), which cannot be written as a fraction of two integers is called a surd.

Whilst numbers like \( \sqrt{2} \) have decimal approximations which can be obtained using a calculator, e.g. \( \sqrt{2} = 1.414 \ldots \), we emphasise that these are approximations, whereas the form \( \sqrt{2} \) is exact.

2. Writing surds in equivalent forms

It is often possible to write surds in equivalent forms. To do this you need to be aware that

\[
\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}
\]

However, be warned that \( \sqrt{a + b} \) is not equal to \( \sqrt{a} + \sqrt{b} \).

For example \( \sqrt{148} \) can be written

\[
\sqrt{3 \times 16} = \sqrt{3} \times \sqrt{16} = 4\sqrt{3}
\]

Similarly, \( \sqrt{60} \) can be written

\[
\sqrt{4 \times 15} = \sqrt{4} \times \sqrt{15} = 2\sqrt{15}
\]

3. Applications

Surds arise naturally in a number of applications. For example, by using Pythagoras’ theorem we find the length of the hypotenuse of the triangle shown below to be \( \sqrt{2} \).

\[
\begin{array}{c}
1 \\
\end{array}
\begin{array}{c}
\sqrt{2} \\
1 \\
\end{array}
\]
Surds arise in the solution of quadratic equations using the formula. For example the solution of \( x^2 + 8x + 1 = 0 \) is obtained as

\[
x = \frac{-8 \pm \sqrt{8^2 - 4(1)(1)}}{2} = \frac{-8 \pm \sqrt{60}}{2} = \frac{-8 \pm \sqrt{4 \times 15}}{2} = \frac{-8 \pm 2\sqrt{15}}{2} = -4 \pm \sqrt{15}
\]

This answer has been left in surd form.

**Exercises**

1. Write the following in their simplest forms.
   a) \(\sqrt{63}\), b) \(\sqrt{180}\).

2. By multiplying numerator and denominator by \(\sqrt{2} + 1\) show that

\[
\frac{1}{\sqrt{2} - 1} \quad \text{is equivalent to} \quad \sqrt{2} + 1
\]

The process of rewriting a fraction in this way so that all surds appear in the numerator only, is called **rationalisation**.

3. Rationalise the denominator of a) \(\frac{1}{\sqrt{2}}\), b) \(\frac{1}{\sqrt{5}}\).

4. Simplify \(\sqrt{18} - 2\sqrt{2} + \sqrt{8}\).

**Answers**

1. a) \(3\sqrt{7}\), b) \(6\sqrt{5}\).

3. a) \(\sqrt{2}\), b) \(\sqrt{5}\).

4. \(3\sqrt{2}\).