Logarithms - changing the base

Sometimes it is necessary to find logarithms to bases other than 10 and e. For example, logarithms to the base 2 are used in communications engineering. Your calculator can still be used but you need to apply a formula for changing the base. This leaflet gives this formula and shows how to use it.

A formula for change of base

Suppose we want to calculate a logarithm to base 2. The formula states

\[ \log_2 x = \frac{\log_{10} x}{\log_{10} 2} \]

So we can calculate base 2 logarithms using base 10 logarithms obtained using a calculator.

Examples

\[ \log_2 36 = \log_{10} 36 \]
\[ \log_{10} 2 = 1 \]
\[ 0.301030 \]
\[ \log_2 36 = \frac{\log_{10} 36}{\log_{10} 2} = 5.1699 \text{ (correct to 4 d.p.)} \]
\[ \log_2 64 = \frac{\log_{10} 64}{\log_{10} 2} = \frac{1.806180}{0.301030} = 6 \]

Check these for yourself. More generally, for bases \( a \) and \( b \),

\[ \log_a x = \frac{\log_b x}{\log_b a} \]

In particular, by choosing \( b = 10 \) we find

\[ \log_a x = \frac{\log_{10} x}{\log_{10} a} \]

Use this formula to check that \( \log_{20} 100 = 1.5372 \text{ (correct to 4 d.p.)} \).

Exercises

1. Find, correct to 3 decimal places, (a) \( \log_2 15 \), (b) \( \log_2 56.25 \), (c) \( \log_3 16 \).
2. By writing the expression in logarithmic form, find the number \( x \) such that \( 2^x = 3.6 \).

Answers

1. (a) 3.907 (3 d.p.), (b) 5.814 (3 d.p.), (c) 2.524 (3 d.p).
2. \( \log_2 3.6 = x \), and so \( x = 1.848 \text{ (3 d.p.)} \).