Factorising quadratics

You will have seen before that expressions like \((x + 2)(x + 3)\) can be expanded to give the quadratic expression \(x^2 + 5x + 6\). Like many processes in mathematics, it is useful to be able to go the other way. That is, starting with the quadratic expression \(x^2 + 5x + 6\), can we carry out a process which will result in the form \((x + 2)(x + 3)\)? This process is called **factorising the quadratic expression**. This leaflet describes this process. Special cases known as **complete squares** and the **difference of two squares** are dealt with on separate leaflets.

**Factorising quadratics**

To learn how to factorise let us study again the previous example when the brackets were multiplied out from \((x + 2)(x + 3)\) to give \(x^2 + 5x + 6\).

\[
(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6
\]

Clearly the number 6 in the final answer comes from **multiplying** the numbers 2 and 3 in the brackets. This is an important observation. The term \(5x\) comes from **adding** the terms 3\(x\) and 2\(x\).

So, if we were to begin with \(x^2 + 5x + 6\) and we were going to reverse the process we need to look for two numbers which add to give 5 and multiply to give 6. What are these numbers? Well, we know that they are 3 and 2, and you will learn with practice to find these simply by inspection. We can set the calculation out as follows. Start with a pair of empty brackets.

\[
x^2 + 5x + 6 = (\text{ })(\text{ })
\]

insert an \(x\) in each

\[
= (x )(x )
\]

these will multiply to give the required \(x^2\)

\[
= (x + 2)(x + 3)
\]

these numbers multiply to give 6 and add to give 5

The answer should always be checked by multiplying-out the brackets again!

**Example**

Factorise the quadratic expression \(x^2 - 7x + 12\).

Starting as before we write

\[
x^2 - 7x + 12 = (\text{ })(\text{ })
\]

and we look for two numbers which add together to give \(-7\) and which multiply together to give 12. The two numbers we seek are \(-3\) and \(-4\) because

\[
-3 \times -4 = 12, \quad \text{and} \quad -3 + -4 = -7
\]

So

\[
x^2 - 7x + 12 = (x - 3)(x - 4)
\]

Once again, note that the answer can be checked by multiplying-out the brackets again. The alternative, equivalent form \((x - 4)(x - 3)\), is also correct.
Factorise Example

( and try, by inspection, to determine the contents of the brackets. There is no point writing \((-\ )(-\ )\) because the two \(x\) terms would multiply to give \(x^2\), and in this example we are looking for \(3x^2\). So try

\[3x^2 + 5x - 2 = (3x + 2)(x - 1)\]

which will certainly generate the term \(3x^2\). The constant term \(-2\) can be generated from the numbers \(-2\) and \(1\), or alternatively \(-1\) and \(2\). So, we are led to consider the following combinations

\[(3x - 2)(x + 1), \quad (3x + 1)(x - 2), \quad (3x - 1)(x + 2), \quad (3x + 2)(x - 1)\]

all of which generate the correct term in \(x^2\) and the correct constant term. However, only one of these generates the correct \(x\) term, \(5x\). By inspection we find

\[3x^2 + 5x - 2 = (3x - 1)(x + 2)\]

Example

Factorise \(2x^2 + 5x - 7\).

To generate the term \(2x^2\) we can write

\[2x^2 + 5x - 7 = (2x + 7)(x - 1)\]

To generate the constant term \(-7\) we need two numbers which multiply together to give \(-7\). Recognise that to produce a negative result one factor must be positive and one must be negative. We are led to consider \(-7\) and \(1\), or alternatively \(-1\) and \(7\). So, we consider the following combinations

\[(2x - 7)(x + 1), \quad (2x + 1)(x - 7), \quad (2x - 1)(x + 7), \quad (2x + 7)(x - 1)\]

By inspection the correct factorisation is \(2x^2 + 5x - 7 = (2x + 7)(x - 1)\).

Exercises

1. Factorise the following.
   a) \(x^2 + 8x + 15\) b) \(x^2 + 10x + 24\) c) \(x^2 + 9x + 8\) d) \(x^2 + 9x + 14\)
   e) \(x^2 + 15x + 36\) f) \(x^2 + 2x - 3\) g) \(x^2 + 2x - 8\) h) \(x^2 + x - 20\)

Quadratic expressions where the coefficient of \(x\) is not 1

Let us try to factorise the expression \(3x^2 + 5x - 2\). We write, as before,

\[3x^2 + 5x - 2 = (\quad)(\quad)\]

and try, by inspection, to determine the contents of the brackets. There is no point writing \((-\ )(-\ )\) because the two \(x\) terms would multiply to give \(x^2\), and in this example we are looking for \(3x^2\). So try

\[3x^2 + 5x - 2 = (3x + 2)(x - 1)\]

Exercise

Factorise \(2x^2 + 5x - 7\).

To generate the term \(2x^2\) we can write

\[2x^2 + 5x - 7 = (2x + 7)(x - 1)\]

To generate the constant term \(-7\) we need two numbers which multiply together to give \(-7\). Recognise that to produce a negative result one factor must be positive and one must be negative. We are led to consider \(-7\) and \(1\), or alternatively \(-1\) and \(7\). So, we consider the following combinations

\[(2x - 7)(x + 1), \quad (2x + 1)(x - 7), \quad (2x - 1)(x + 7), \quad (2x + 7)(x - 1)\]

By inspection the correct factorisation is \(2x^2 + 5x - 7 = (2x + 7)(x - 1)\).

Exercises

2. Factorise the following.
   a) \(2x^2 + 11x + 5\) b) \(3x^2 + 19x + 6\) c) \(3x^2 + 17x - 6\) d) \(6x^2 + 7x + 2\)
   e) \(7x^2 - 6x - 1\) f) \(12x^2 + 7x + 1\) g) \(8x^2 + 6x + 1\) h) \(8x^2 - 6x + 1\)

Answers

1. a) \((x + 3)(x + 5)\) b) \((x + 4)(x + 6)\) c) \((x + 1)(x + 8)\) d) \((x + 2)(x + 7)\)
   e) \((x + 3)(x + 12)\) f) \((x + 3)(x - 1)\) g) \((x + 4)(x - 2)\) h) \((x + 5)(x - 4)\)

2. a) \((2x + 1)(x + 5)\) b) \((3x + 1)(x + 6)\) c) \((3x - 1)(x + 6)\) d) \((2x + 1)(3x + 2)\)
   e) \((7x + 1)(x - 1)\) f) \((3x + 1)(4x + 1)\) g) \((2x + 1)(4x + 1)\) h) \((2x - 1)(4x - 1)\)